 Basic

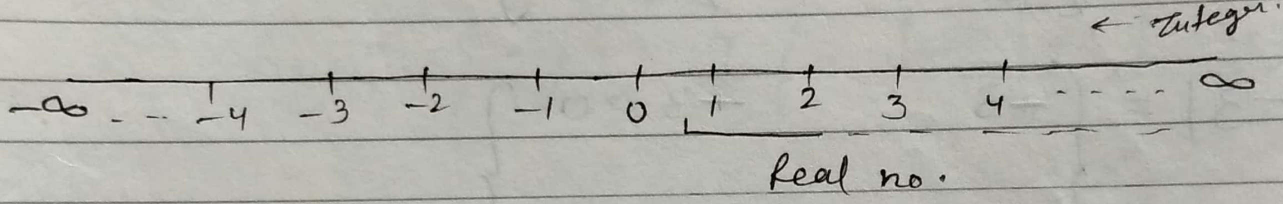
01/05/17 Maths

$*$ Inequality sign Changed in condition

$1 < 2$

$2 > 1$ $\quad$ $-1 > -2$ $\quad$ $\frac{1}{1} > \frac{1}{2}$

Interchanging $\quad$ Negative multiplicative $\quad$ resiprocal

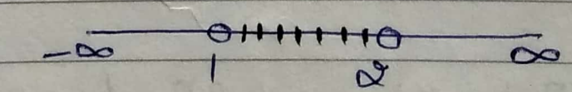


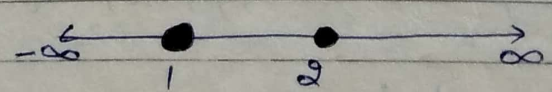
Number line or real line (R).

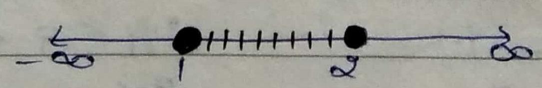
$*$ $*$ $x \in R$ (x belongs to R)

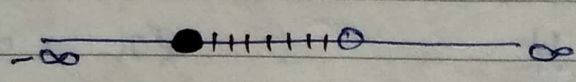
$x^2 \geq 0$ (x greater than and equal to 0)

(x^2 is not -ve)

$*$ $x \in (1,2)$  i.e $1 < x < 2$

$x \in \{1,2\}$  i.e $x = 1, 2$

$x \in [1,2]$ 

$x \in [1,2)$ 

* Moduls :

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$|2| = \begin{cases} 2 & 2 > 0 \\ -2 & 2 < 0 \end{cases}$$

$$|-3| = \begin{cases} 3 & 3 > 0 \\ +3 & 3 < 0 \end{cases}$$

$$|x-1| = \begin{cases} x-1 & x \geq 1 \\ -(x-1) & x < 1 \end{cases}$$

$$|3x-2| = \begin{cases} 3x-2 & x \geq 2/3 \\ -(3x-2) & x < 2/3 \end{cases}$$

* $|2x-3| = 1$ find x.

$2x - 3 = \pm 1$

$2x - 3 = 1$

$2x = 1 + 3$

$x = \frac{4}{2}$

$x = 2$

$2x - 3 = -1$

$2x = -1 + 3$

$2x = 2$

$x = \frac{2}{2}$ $\quad x = 1$

Ans $= \{1, 2\}$

* $|x-1| + |x-3| = 5$

$-\infty < x < 1$	$1 \le x < 3$	$3 \le x < \infty$
$-(x-1) + \cdot -(x-3) = 5$	$(x-1) + \cdot -(x-3) = 5$	$(x-1) + (x-3) = 5$
$-x+1-x+3 = 5$	$x-1-x+3 = 5$	$2x - 4 = 5$
$-2x = 1$	$2 = 5$	$2x = 5+4$
$x = \frac{-1}{2}$	ϕ	$2x = 9$
		$x = \frac{9}{2}$

$-\infty$ ... 1 ... 3 ... ∞

Ans : $\left\{ \frac{-1}{2}, \frac{9}{2} \right\}$

$-ve$	$+ \cdot -$	$+ve$

* $|x-1| = \begin{cases} x-1 & x \ge 1 \\ -(x-1) & x < 1 \end{cases}$

$|x-3| = \begin{cases} x-3 & x \ge 3 \\ -(x-3) & (x < 3) \end{cases}$

* **Rules :**

1. If $|x| < 1 \Rightarrow -1 < x < 1$ i.e $x \in (-1, 1)$
2. If $|x| \le 1 \Rightarrow -1 \le x \le 1$ i.e $x \in [-1, 1]$
3. $|x-4| \Rightarrow |4-x|$
4. If $|x| > 1 \Rightarrow x < -1$ $x > 1$ i.e $x \in (-\infty, -1) \cup (1, \infty)$.

5. $a^2 < x^2 < b^2 \Rightarrow a < |x| < b \Rightarrow x \in (-b, -a) \cup (a, b)$

6. If $9 \le x^2 \le 16$ or $3 \le |x| \le 4$

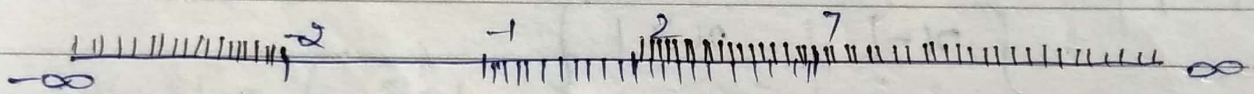
$$\Rightarrow x \in [-4, -3] \cup [3, 4]$$

* Solve $|x-3| \le 4$ & $|x| > 2$

$-4 \le x - 3 \le 4$ $\qquad$ $x > 2$ $x < -2$

$-4 + 3 \le x - 3 + 3 \le 4 + 3$

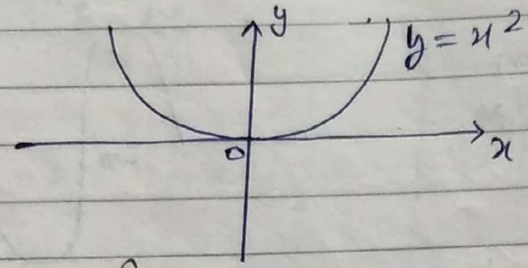
$-1 \le x \le 7$



Common Ans: $2 < x \le 7$

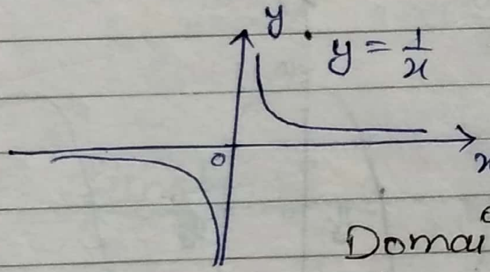
* Basic algebraic function :

(i) $y = x^2$



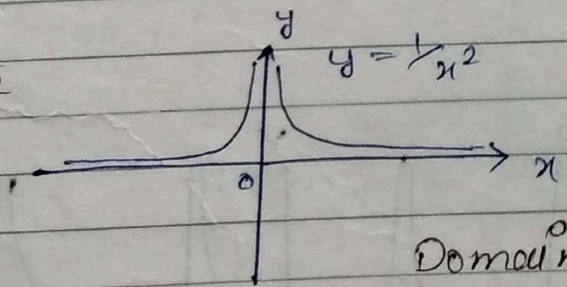
Domain : R
Range : $R^+ \cup \{0\}$ or $[0, \infty)$

(ii) $y = \frac{1}{x}$



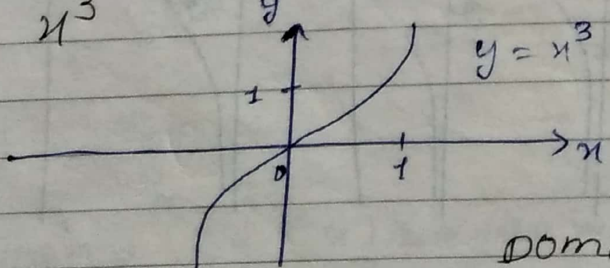
Domain : $R - \{0\}$ or R_0
Range : $R - \{0\}$

(iii) $y = \frac{1}{x^2}$



Domain : R_0
Range : R^+ or $(0, \infty)$

(iv) $y = x^3$



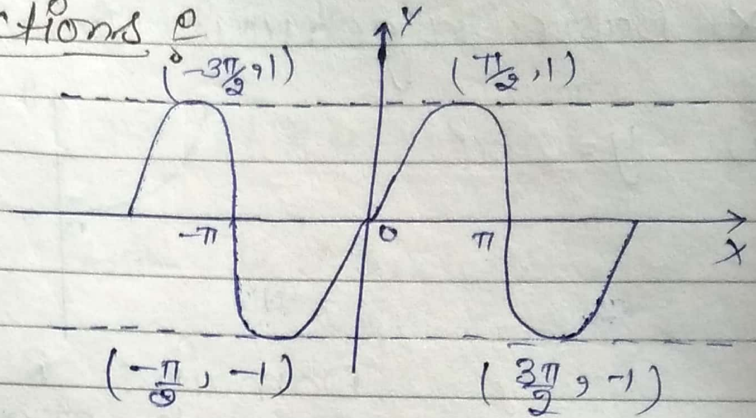
Domain : R
Range : R.

$R = [-\infty, \infty]$

* Trigonometric functions :

(i) Sine function

$f(x) = \sin x$

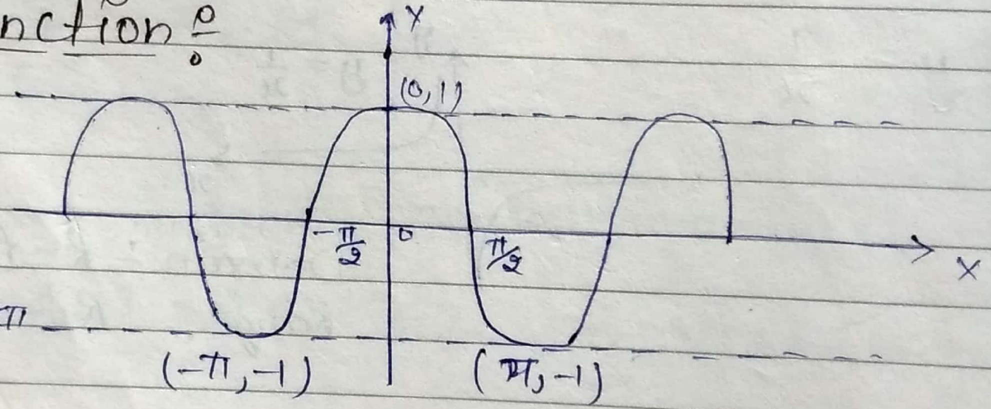


Domain : R

Range : $[-1, 1]$, Period 2π

(ii) Cosine function :

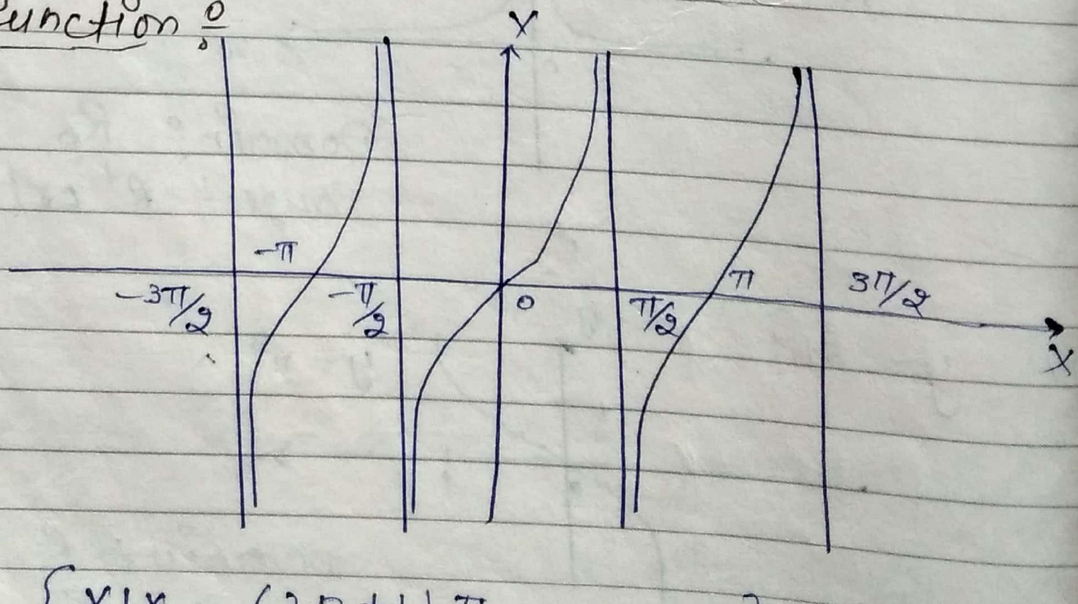
$f(x) = \cos x$



Domain : R

Range : $[-1, 1]$, Period 2π

(iii) Tangent function :



$f(x) = \tan x$

Domain : $R - \left\{ x \mid x = \frac{(2n+1)\pi}{2},\ n \in I \right\}$

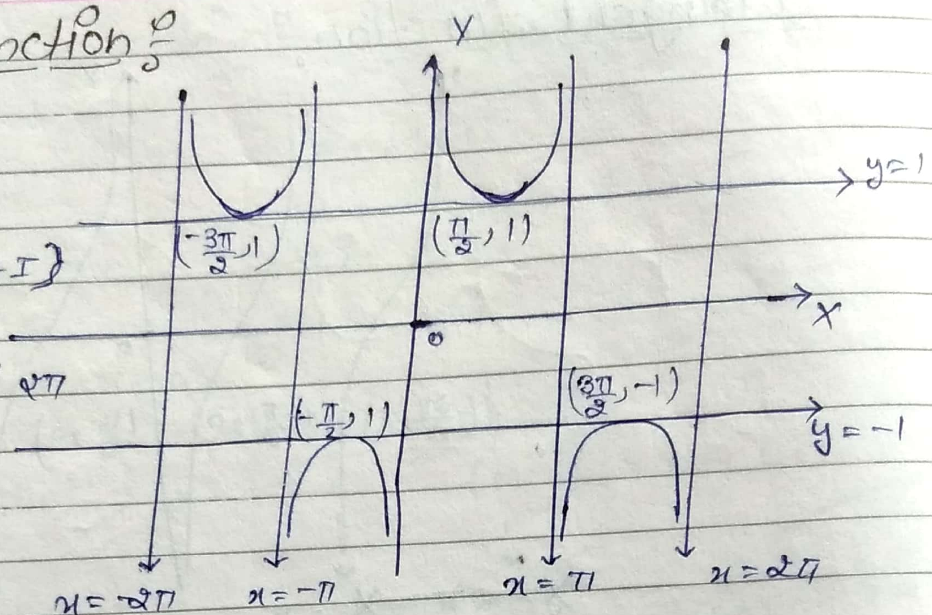
Range : R, Period π

(iv) Cosecant function:

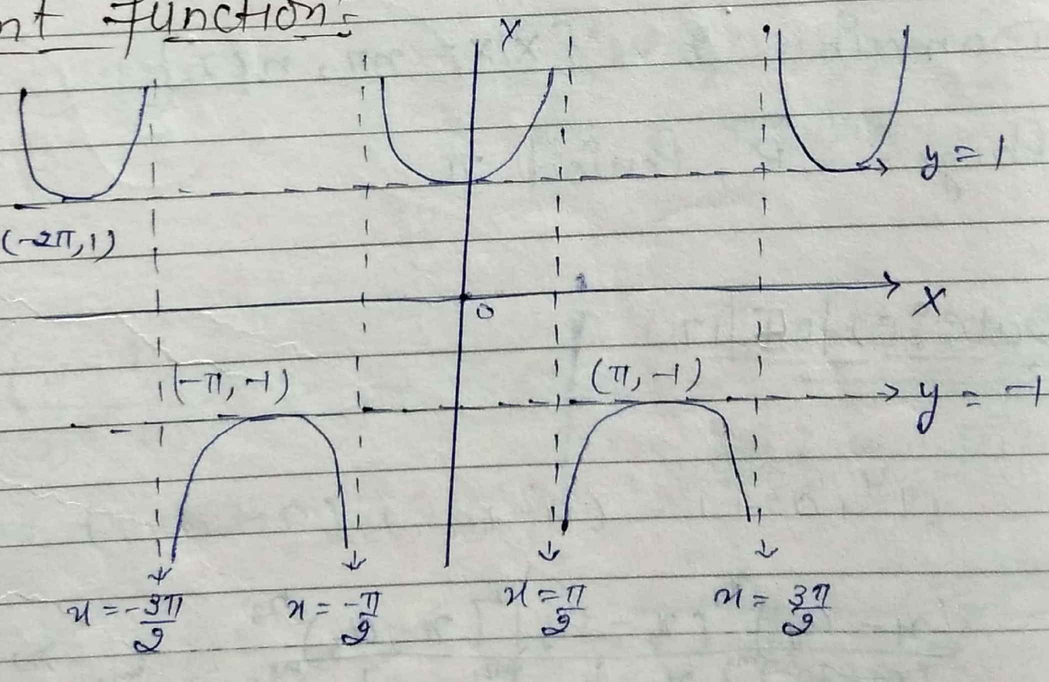
$f(x) = \operatorname{cosec} x$

Domain: $R - \{x \mid x = n\pi, n \in I\}$

Range: $R - (-1, 1)$, Period 2π



(v) Secant function:

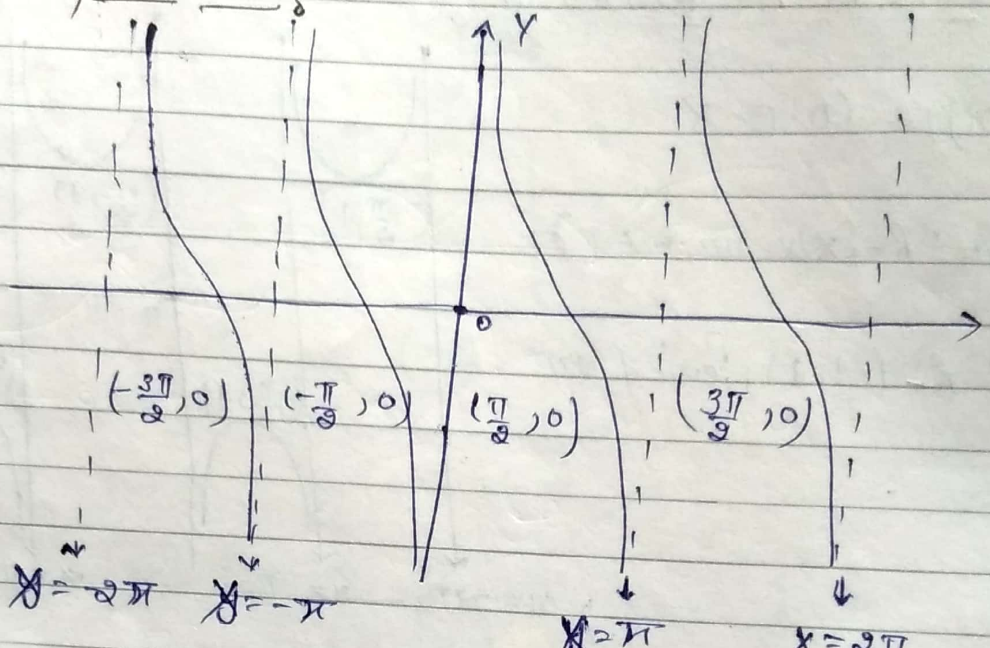


$f(x) = \sec x$

→ Domain = $R - \{x \mid x = (2n+1)\pi/2 : n \in I\}$

Range: $R - (-1, 1)$, Period 2π.

(vi) Cotangent function:



$f(x) = \cot x$

→ Domain : $R - \{x \mid x = n\pi, n \in I\}$

Range : R, Period π.

Date: 02/05/17

$$a^4 + a^2 + 1 = (a^2 + a + 1)(a^2 - a + 1).$$

$$\frac{(x-a_1)^{n_1}(x-a_2)^{n_2}(x-a_3)^{n_3}}{(x-b_1)^{m_1}(x-b_2)^{m_2}(x-b_3)^{m_3}(x-b_4)^{m_4}} >, \geq, <, \leq = 0$$

Rational Inequality
(Wavy Curve Method).

[wavy curve → odd - cross, even - not cross]

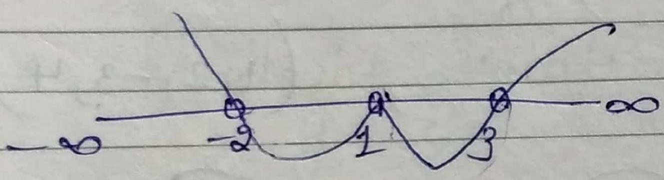
$(x-1) > 0$

* $n_1, n_2 \,--\, m_1, m_2 \,---\, \in N$

* ~~If~~ coff of x in each form is 1

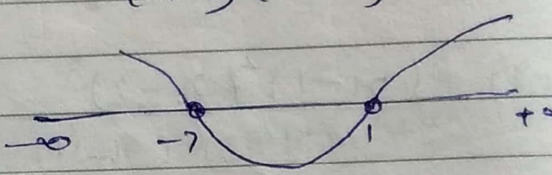
* R.H.S must be zero

$$\frac{(x-1)^2\,(x+2)}{(x-3)^5} > 0$$



Ans: $(-\infty, -2) \cup (3, \infty)$

Que! $(x-1)(x+7) < 0$



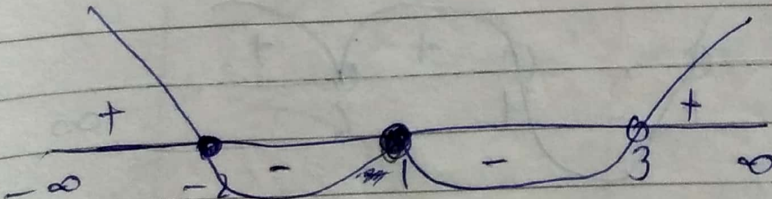
$x \in (-7, 1)$

$(x-1)(x+7) > 0$

Ans $(-\infty, -7) \cup (1, \infty)$.

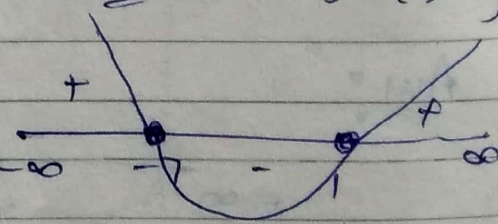
(2) $$\frac{(x-1)^2\,(x+2)}{(x-3)^5} \geq 0$$

Ans = $(-\infty, -2] \cup (3, \infty) \cup \{-2, 1\}$



③ $(x-1)(x+7) \leq 0$

Ans $(-\infty, -7) \cup (1, \infty)$

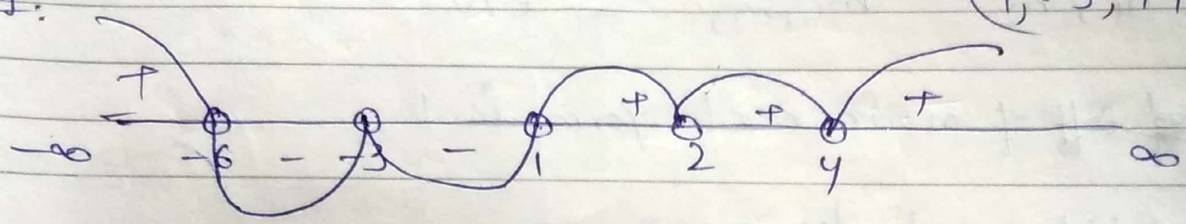


$(-7, 1) \cup \{-7, 1\}$

even = not cross
odd = cross.

Que: $\dfrac{(x-1)^3(x+3)^4(x-4)^2}{(x-2)^6(x+6)} > 0$

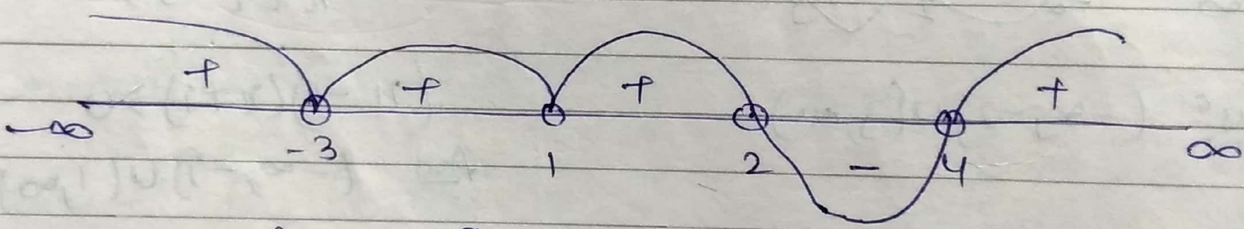
Ans: $(1, -3, +4, 2, -6)$



Ans: $(-\infty, -6) \cup (1,2) \cup (2,4) \cup (4, \infty)$

(2) $\dfrac{(x-1)^2(x-2)^3}{(x+3)^6(x-4)} < 0$

$(1, 2, -3, 4)$

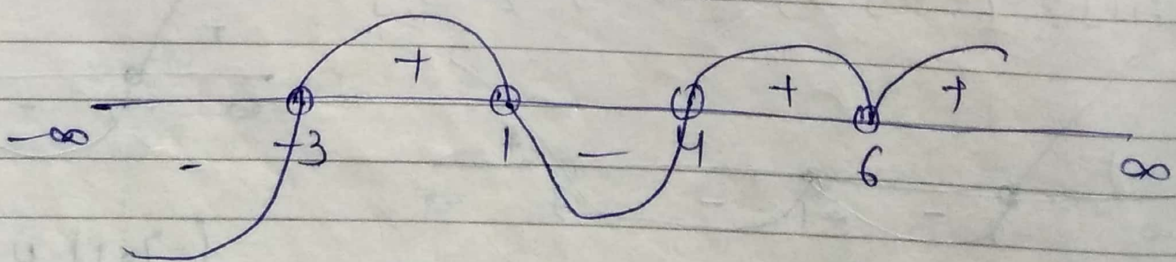


Ans: $(2, 4)$

(3) $\dfrac{(x-1)^3(x-4)^5}{(x-6)^2(x+3)} > 0$

$(1, 4, 6, -3)$

Ans:



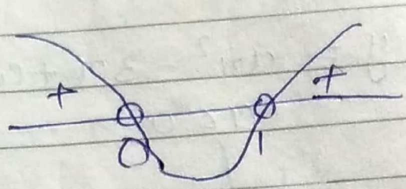
Ans $(-3, 1) \cup (4, 6) \cup (6, \infty)$

Que: $\frac{1}{x} < 1$

$\frac{1}{x} - 1 < 0$

$\frac{1-x}{x} < 0$

$\frac{-1(x-1)}{x-0} < 0 \Rightarrow \frac{x-1}{x-0} > 0$



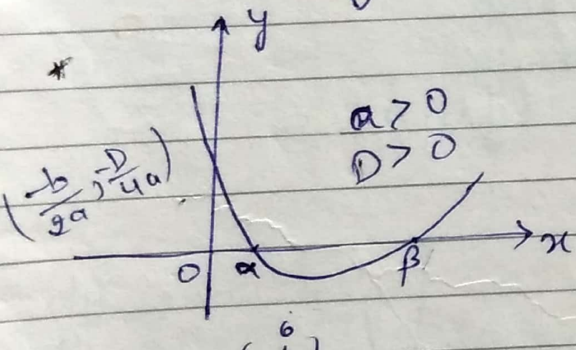
Ans: $x \in (-\infty, 0) \cup (1, \infty)$

* Quadratic equation:

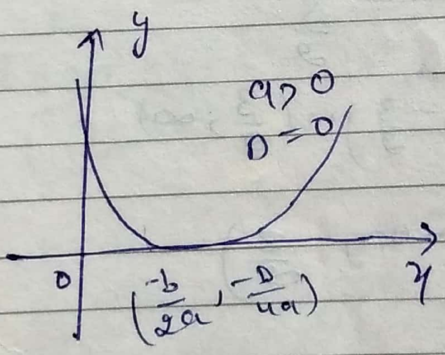
(Parabolet figure)

$y = ax^2 + bx + c$

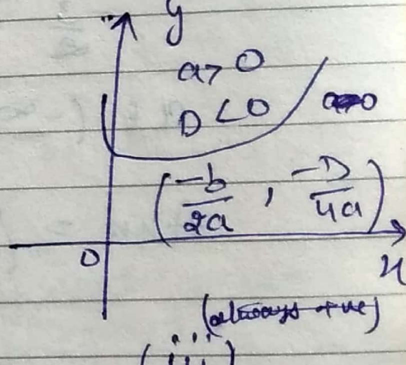
*



(i)

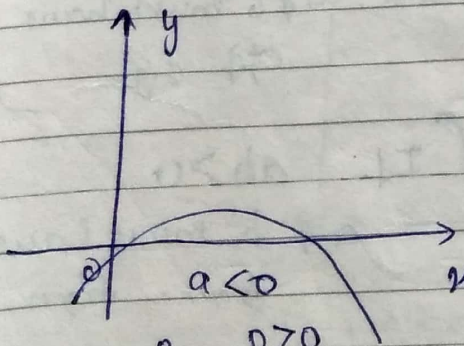


(ii)



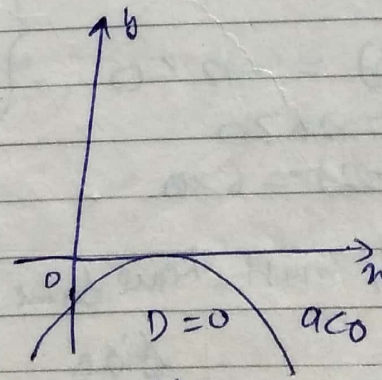
(iii)

Imp



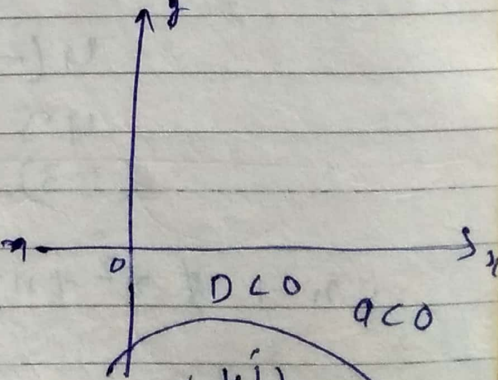
(iv)

$\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$



(v)

$\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$



(vi)

Imp.

(always -ve)

* 12

$D = b^2 - 4ac$

Que: If $ax^2 - 3x + a < 0 \ \forall x$ the find Range.

Sol: $y = ax^2 - 3x + a$

$a < 0 \longrightarrow$

$D < 0$

$D = b^2 - 4ac \quad (-3)^2 - 4 \cdot a \cdot a < 0$

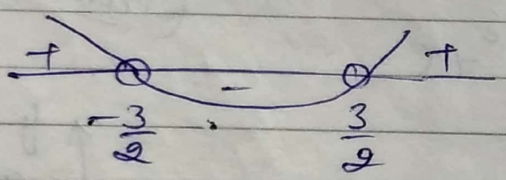
$9 - 4a^2 < 0$

$4a^2 - 9 > 0$

a

$\longrightarrow$

$\left(a - \frac{3}{2}\right)\left(a + \frac{3}{2}\right) > 0$



$a \in \left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$

Ans $= \left(-\infty, -\frac{3}{2}\right)$

Que: $(x-1) \cdot (x^2 + x + 1) > 0$

$4(-3) = -12 < 0$

$4 \cdot 5 = 20 > 0$

$(-3)(-2) = 6 > 0$

$x - 1$ & $x^2 + x + 1$ must have same sign

$x^2 + x + 1 > 0 \ \forall x$

$x - 1 > 0$

$x > 1$

If $a \cdot b < 0$ then a & b must have opp sign.

If $ab > 0 \Rightarrow a$ & b must have same sign.

$y = x^2 + x + 1$

$a = 1 > 0$ (I, II, III)

$D = (1)^2 - 4 \cdot 1 \cdot 1 < 0$

$x^2 + x + 1 > 0 \ \forall x \in R$

Date: 03/05/17

$$y = x^2 - 4x + 9$$

$a = 1$, $b = -4$, $c = 9$

$D = b^2 - 4ac \quad = d < 0$

$(-4)^2 - 4ac$

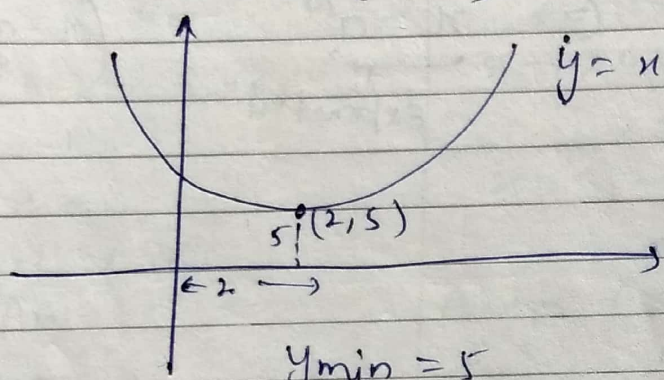
$16 - 4\cdot1\cdot9$

$= -20$

$$\left(\frac{-b}{2a}, \frac{-D}{4a}\right) \Rightarrow \frac{-b}{2\cdot1}, \frac{-D}{4a}$$

$$\frac{-(-4)}{2}, \frac{-(-20)}{4\cdot1} = (2, 5)$$

$$= (2, 5)$$



$y = x^2 - 4x + 9$

$$\boxed{\left(\frac{-b}{2a}, \frac{-D}{4a}\right)}$$

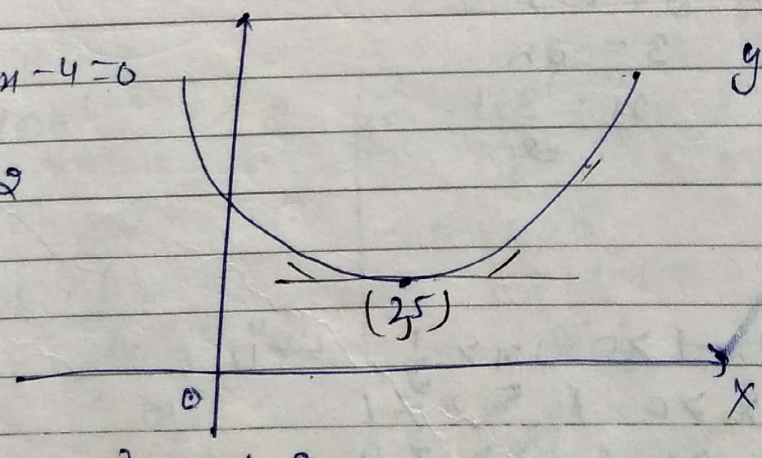
$y_{min} = 5$

$y_{max} = \infty$

range $= [5, \infty)$

$\frac{dy}{dx} = 2x - 4 = 0$

$x = 2$



$y(x=2) = 2^2 - 4\cdot2 + 9$
$= 5$

$y = x^2 - 4x + 9$

$= x^2 - 2\cdot2x + 4 + 5$

$= (x-2)^2 + 5$

formula

$(x-2)^2 = x^2 + - -)$

$x^2 - 2\cdot2x + 4 + 5$

$(x-2)^2 + 5$

$\log_3 2 = 1$

$2 = 3^1$

$\log_a x = N$ — $\log x$ base a

base

defined ~~log~~ base $x>0$ & $a>0,\ a\neq 1$

$\log_a x = N \iff \underbrace{x = a^N}_{\text{Exponettal}}$ (x = axial function)

Que: find $\log_4 8$

Ans: $\log_4 8 = n$

Log $8 = 4^n$

$2^3 = (2^2)^n$

$2^3 = 2^{2n}$

$3 = 2n$

$n = \frac{3}{2}$

$\left[(a^m)^n = a^{mn}\right]$

Que: $\log_{(5-x)}(2x-1)$

$2x-1>0 \Rightarrow x > \frac{1}{2}$ —(i)

$5-x>0$ & $5-x \neq 1$

$x-5>0$ & $5-x\neq 0$

$x-5<0$

(ii)— $\boxed{x<5}$ $\boxed{x\neq 4}$ —(iii)

$\frac{1}{2}<x<5\ (\neq 4)$

Ans $\left(\frac{1}{2}, 5\right) - \{4\}$

$\log_a^x = N$

$x > 1$ — I

$0 < a < 1$ — D

Que: $\log_2^{(x-1)} = 1$

Sol: $x-1 > 0 \Rightarrow x > 1$ —(i)

$x - 1 = 2^1$

$x = 3$ —(ii)

from (i) & (ii)

Ans: 3

(2) $\log_2^{(x-1)} > 1$

$x - 1 > 0 \Rightarrow x > 1$ —(i)

$x - 1 > 2^1$

$x > 3$ —(ii)

Ans: $(3, \infty)$

(3) $\log_2^{(x-1)} < 2$

$x - 1 > 0$

$x - 1 < 2^2$

$x < 5$

(4) $\log_3^{(2x-1)} \leq 2$

$2x - 1 > 0$

$x > \frac{1}{2}$ —(i)

$2x - 1 \leq 3^2$ —(ii)

$2x \leq 10$

$x \leq 5$ —(ii)

Ans $x \in (\frac{1}{2}, 5]$

(5) $\log_{\frac{1}{2}}^{(x-1)} > 1$

$x - 1 > 0$

$x > 1$ —(i)

$x - 1 \cancel{>} \frac{1}{2}$

$x \cancel{>} \frac{3}{2}$ —(ii)

for (i) & (ii)

$1 < x \cancel{>} \frac{3}{2}$

$\log_{1/3}^{(2x-1)} \leq 2$

$2x - 1 > 0$ —(i)

$2x - 1 \geq (\frac{1}{3})^2$ —(ii)