

# SBG STUDY

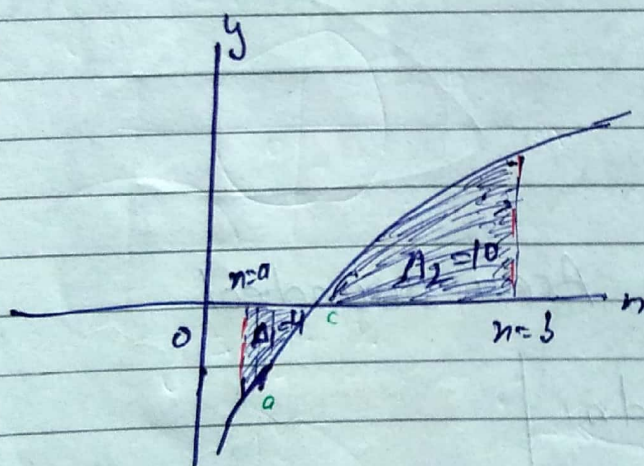
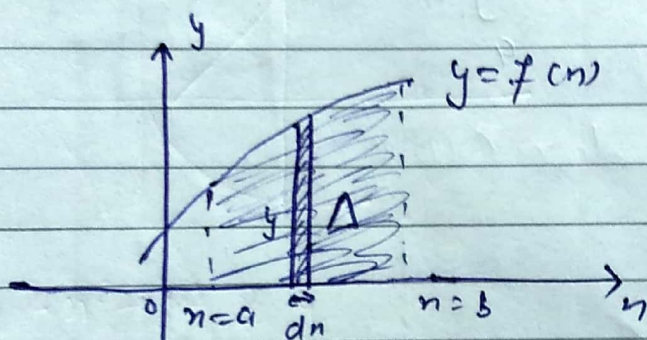
10/08/17

Chapter -

## Area Under the Curve

$$\int_a^b y \, dx = \int_a^b f(x) \, dx = \Delta$$

Algebraic sum of Area bounded by curve  $y = f(x)$  and ordinate  $x = a$  and  $x = b$



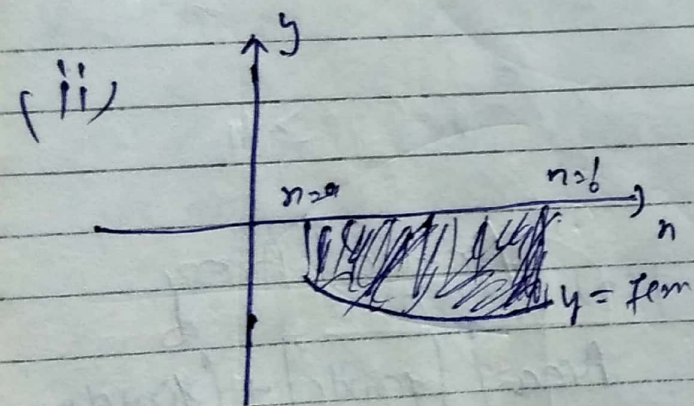
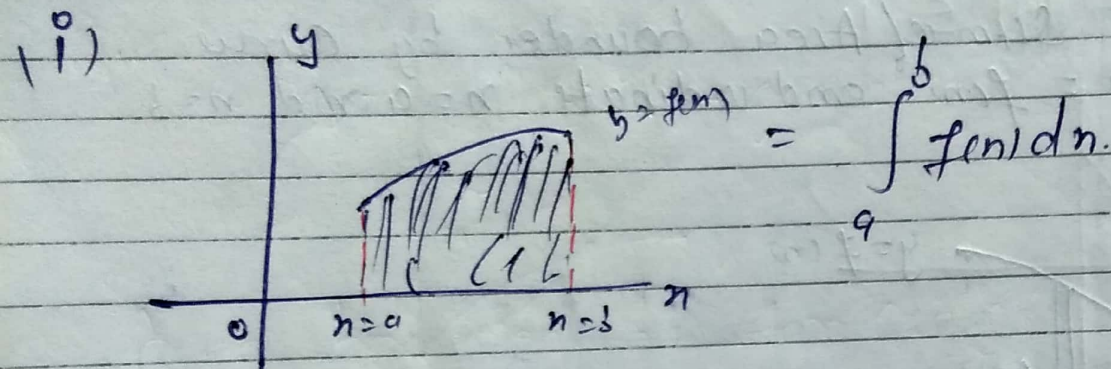
Actual Area

$$\text{Area} = \left| \int_a^c f(x) \, dx \right| + \int_c^b f(x) \, dx$$
$$= 4 + 10$$
$$= 14$$

$$\int_a^b f(x) \, dx = 10 + (-4)$$
$$= 6$$

## \* Formula!

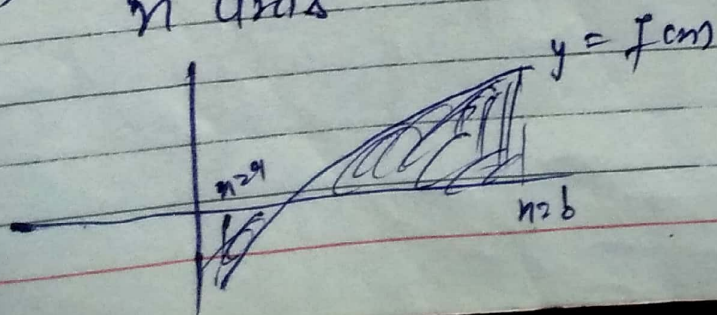
1. If Curve  $y = f(x)$  completely lie above  $x$ -axis then area bounded by curve,  $x$ -axis and lines  $x = a$  and  $x = b$ .



If fig. is this then Area bounded

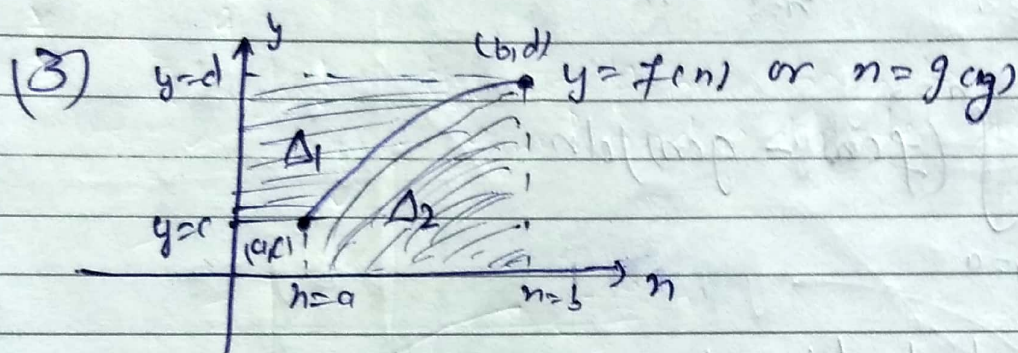
$$\text{Ar. B} = \left| \int_a^b f(x) dx \right|$$

(iii) if path of curve lie above and below  $x$  axis



Then area bounded =

$$A = \left| \int_a^c f(n) dn \right| + \int_c^b f(n) dn$$



$$y = n^2 \Rightarrow n = \sqrt{y}$$

$$f(n) \quad n = g(y)$$

\* Area bounded by Curve  $n = g(y)$ ,  $y$ -axis and the lines  $y=c$  and  $y=d$

$$\Delta_1 = \int_c^d g(y) dy$$

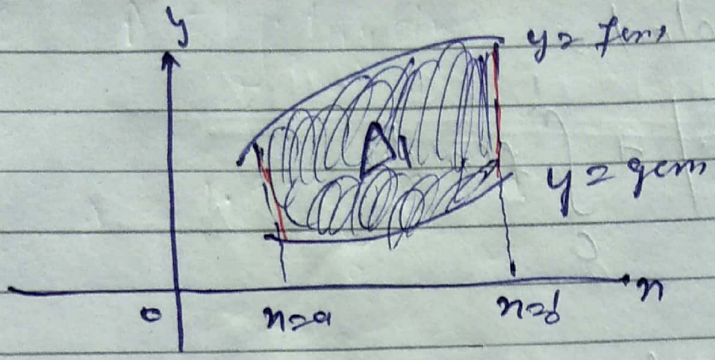
\*

$$\Delta_2 = \int_a^b f(n) dn$$

$$\Delta_1 + \Delta_2 = bd - ac$$

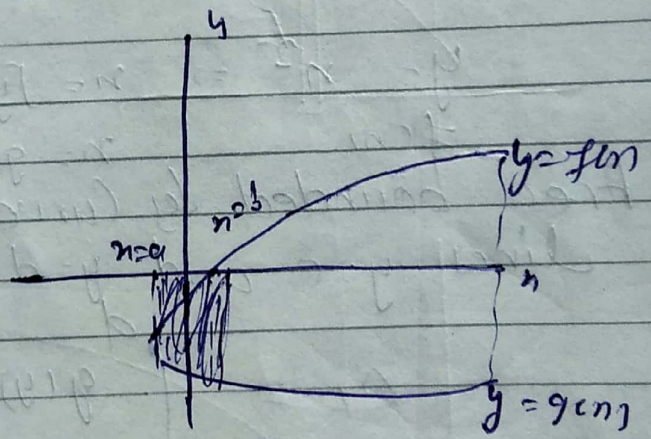
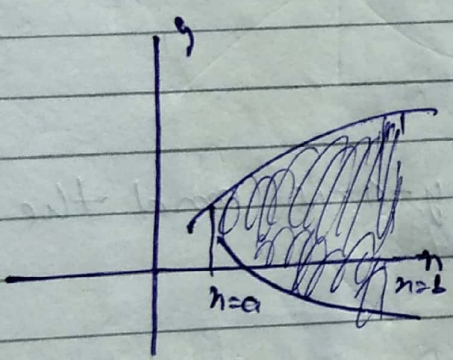
(4)

(4)



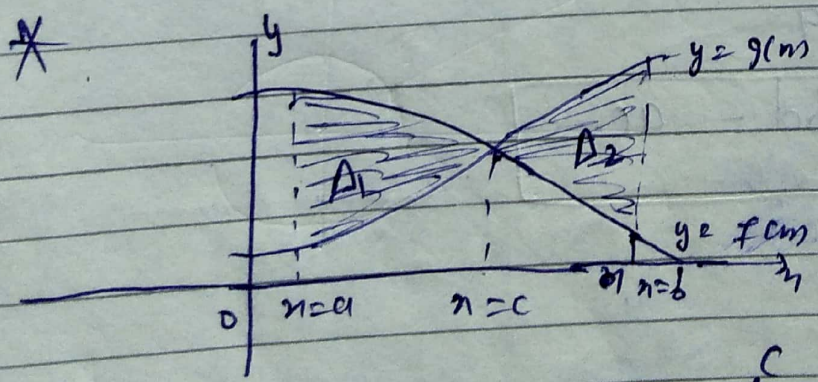
$$\Delta = \int_{x=a}^{x=b} (f(x) - g(x)) dx$$

Area bounded b/w two Curve



same

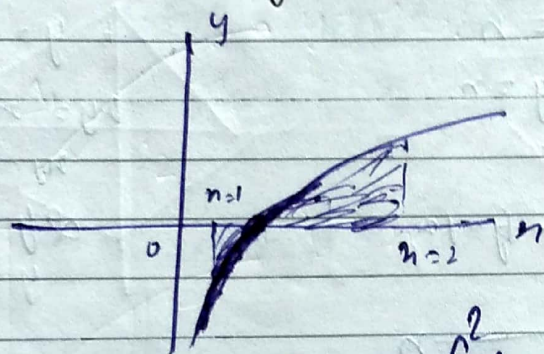
$$\Delta = \int_{x=a}^{x=b} (f(x) - g(x)) dx$$



$$\text{Area bounded} = A_1 + A_2 = \int_a^c (f+g) dx + \int_c^b (g-f) dx$$

$$\text{or } = \int_a^b |f-g| dx$$

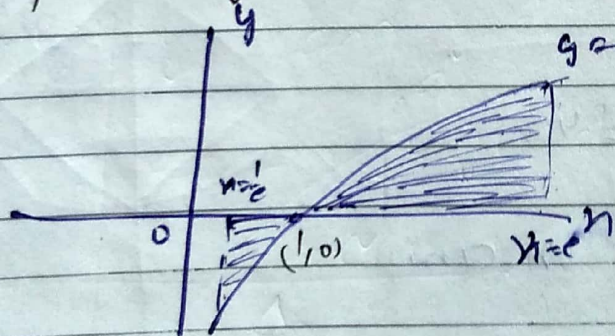
Ques! find AB  $y = \ln x$ ,  $x=1$ ,  $x=2$  and  $x$  axis.



$$= \int_{x=1}^{x=2} f(x) = \int_1^2 dx$$

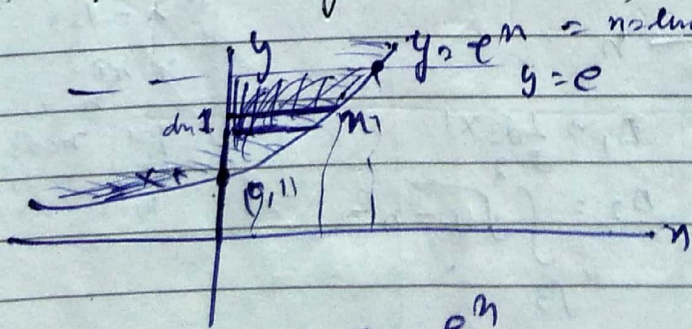
$$A = \int_1^2 \ln x dx$$

Q. find AB  $y = \ln x$   $x = \frac{1}{e}$ ,  $x = e$ ,  $x$ -axis



$$A = \int_{1/e}^1 \ln x dx + \int_1^e \ln x dx$$

Ques. find AB  $y = e^x$ ,  $x=0$ ,  $y=e$ ,  $x > 0$



$$= A = \int_0^1 e^x dx =$$

$$= A = \int_0^1 (e - e^x) dx$$

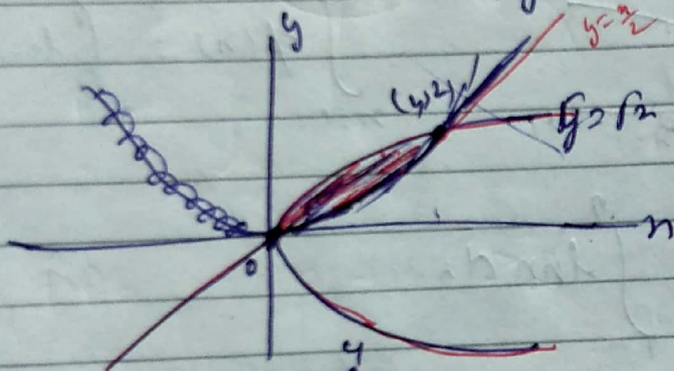
$$e = e^x$$

$$x = 1$$

(M-2)  $A = \int_1^e \ln y \, dy$

$A = 1 \cdot e - \int_0^1 e^{1/n} \, dn$

Que! FAB =  $y^2 = n$   
 $\& y = n$



$A = \int_0^1 (\sqrt{n} - \frac{n}{2}) \, dn$

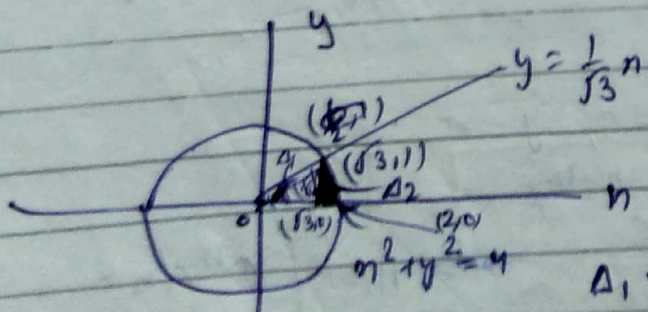
$y = \sqrt{n}$   
 $2y = n$   
 $ny = n$   
 $\& y = n$   
 $y = \frac{n}{2}$

$n=1$   
 $y=1$   
 $n=2$   
 $y=1$   
 $n=3$   
 $y=1$

Q. FAB =  $n^2 + y^2 = 4$

$n = \sqrt{3}y$

first quad. n axis



$A_1 = \frac{1}{2} \times \sqrt{3} \times 1$   
 $A_2 = \int_{\sqrt{3}}^2 \sqrt{4-n^2} \, dn$

$A = \int$

$n = \sqrt{3}y$

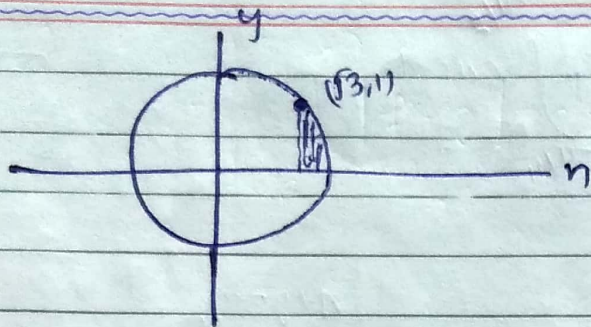
$\Rightarrow \lambda = \sqrt{3}y$

$n \, x \, y =$

$y=1 \quad n=\sqrt{3}$

$y=2 \quad n=2\sqrt{3}$

$n=2\sqrt{3}$



$$A = \int_{\sqrt{3}}^2 \sqrt{4-n^2} dn$$

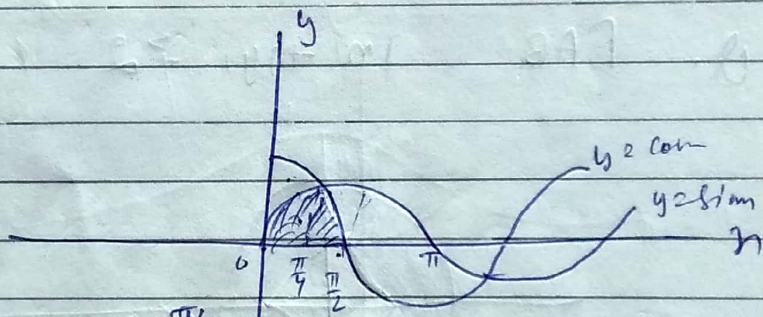
M-2

$$2\pi = \pi(2)^2$$

$$\therefore r^2 = \frac{4\pi}{2\pi}$$

$$\therefore \frac{\pi}{6} = \frac{\pi}{6} \times \frac{4\pi}{2\pi} = \frac{\pi}{3} \quad \text{Ans.}$$

\* Q. FAB  
 $y = \sin n$   
 $y = \cos n$   
 $n = 0$   
 $n = \frac{\pi}{2}$

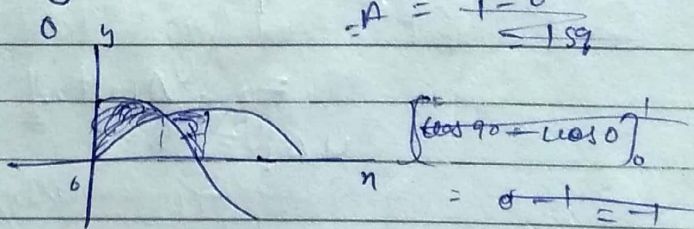


$$A = \int_0^{\pi/2} \sin n dn$$

$$A = \int_0^{\pi/4} (\cos n - \sin n) dn + \int_{\pi/4}^{\pi/2} (\sin n - \cos n) dn$$

$$A = \int_0^{\pi/2} \sin n dn$$

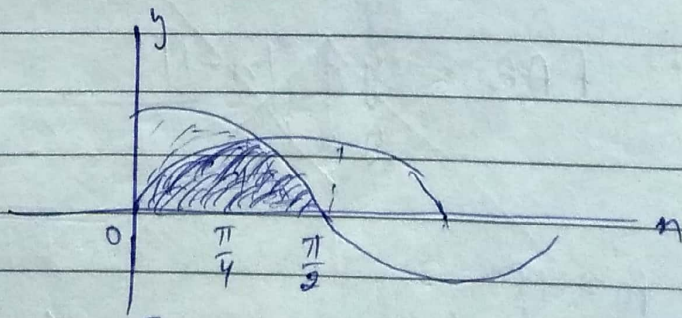
$$= A = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$



$$\left[ \cos n + \sin n \right]_0^{\pi/2} = 0 + 1 - 1 - 0 = 0$$

Q. FAB

$y = \sin n$   
 $y = \cos n$   
 $n = 0$   
 $n = \frac{\pi}{2}$   
 $n = \text{any}$

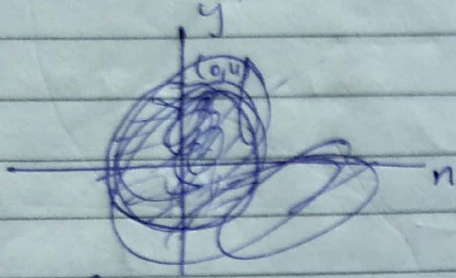


$$A = \int_0^{\pi/4} (\cos n - \sin n) dn + \int_{\pi/4}^{\pi/2} (\sin n - \cos n) dn$$

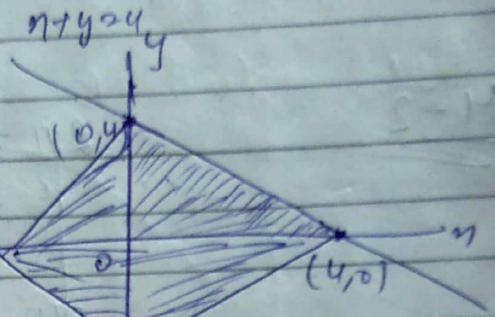
$$A = \int_0^{\pi/4} \sin x \, dx + \int_{\pi/4}^{\pi/2} \cos x \, dx$$

Que!

$$|x| + |y| \leq 4$$

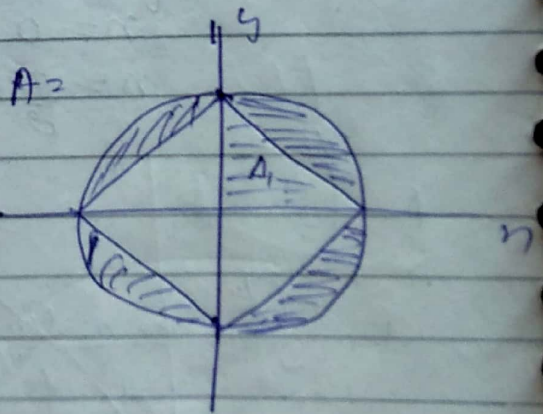
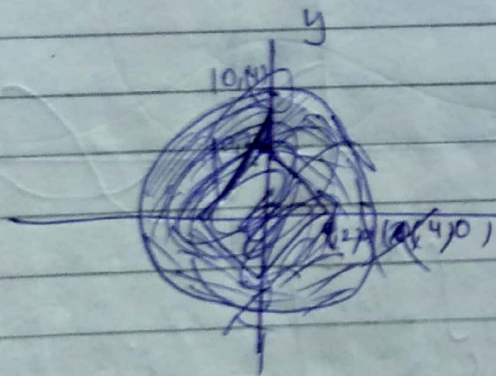


$$A = \int_0^{\dots}$$



$$A = \frac{1}{2} \times 4 \times 4 = 2 \times 4 = 8 \text{ Ans}$$

Q. FAB =  $|x| + |y| \geq 2$  &  $x^2 + y^2 \leq 4$

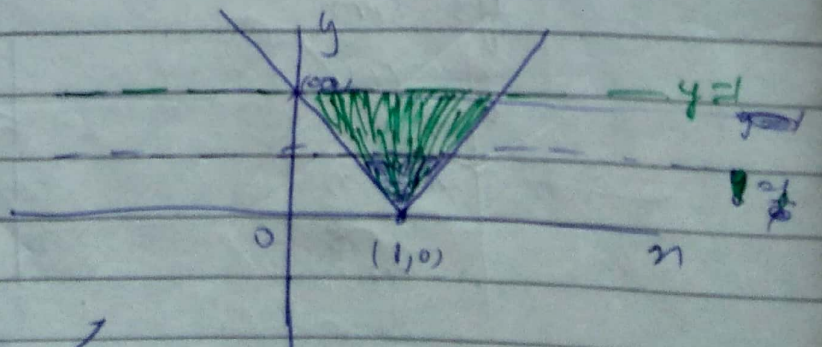


$$\begin{aligned} &= \Delta_1 = \frac{1}{2} \cdot 2 \cdot 2 = 2 \\ 4\Delta_1 &= 8 \\ &= \pi(2)^2 - 4\Delta_1 \\ &= 4\pi - 8 \end{aligned}$$

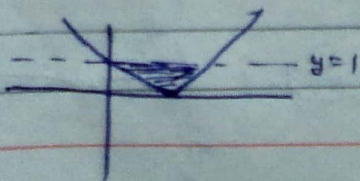
Que!

$$FAB = y = |x-1|$$

$$y = 1$$

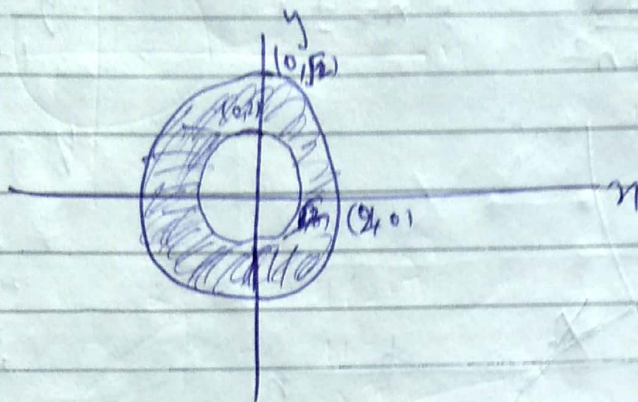


$$A = \frac{1}{2} \times 2 \times 1 = 1$$





Q. FAB:  $2 \leq x^2 + y^2 \leq 4$   $x^2 + y^2 \geq 2$   $x^2 + y^2 \leq 4$

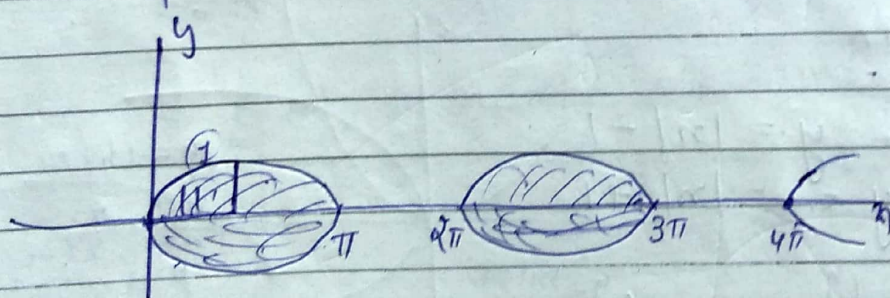
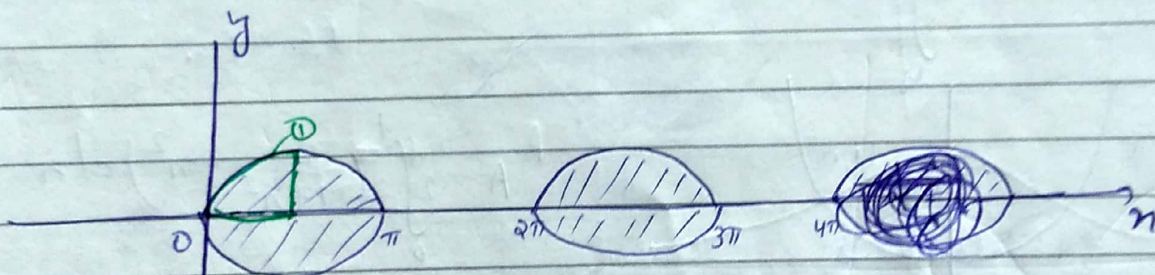


$$A = \frac{\int_{0}^{2\pi} r^2 dr}{2} = 2$$

$$A = \pi(2^2) - \pi(\sqrt{2})^2$$

$$A = 4\pi - 2\pi = 2\pi$$

Q. FAB:  $|y| = 8 \sin x$   $x \in (0, 4\pi)$



$$A = \int_0^{\pi} 8 \sin x dx + \int_{2\pi}^{3\pi} 8 \sin x dx = 8A$$

$\cos 5\pi - \cos 0$   
 $\cos 3\pi - \cos 2\pi$

$$\int_0^{\pi/2} \sin x = 1$$

$$80 = 1 \times 8 = 8$$

Q. FAB =  $y = n^2$

$y = \frac{2}{1+n^2} = n = \pm 1$

$y = \frac{2}{1+n^2}$

$y = \frac{2}{1+n^2}$

$n=1, y=1$

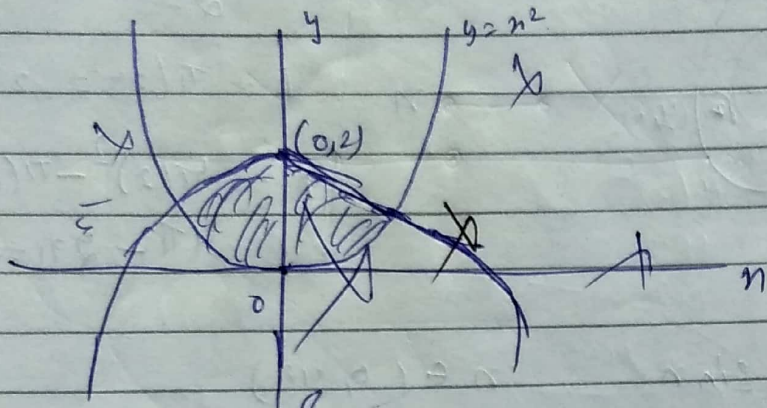
$n=2, y=\frac{2}{5}=0.4$

$n=3, y=\frac{2}{10}$

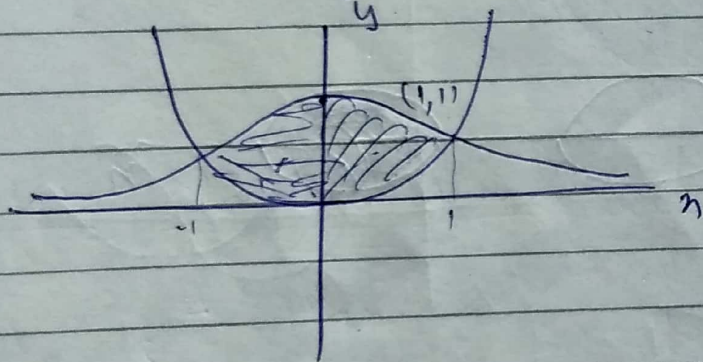
$n=5, y=\frac{2}{26}$

$n=1, y=1$

$n=1, y=1$



A =



$A = 2 \int_0^1 \left( \frac{2}{1+n^2} - n^2 \right) dn$

Q. FAB =  $y = |n| - 1$

$y = -|n| + 1$

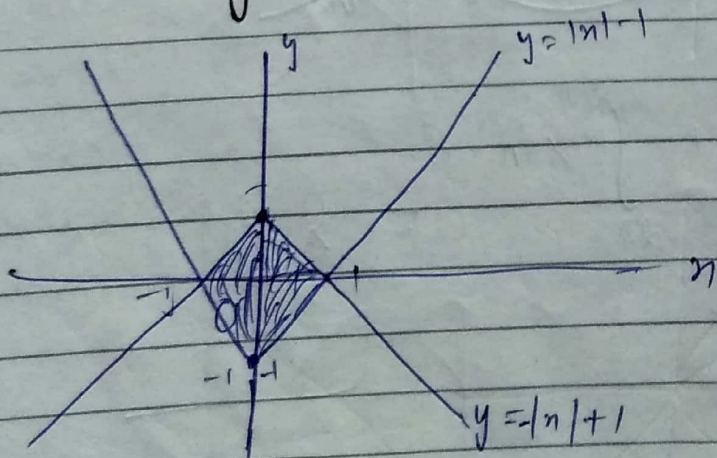
$y = -|n| + 1$

$n=1, y=0$

$n=2, y=-1$

$n=3, y=-2$

$n=0, y=1$



$A = \int_{-1}^1 (|n| - 1 - (-|n| + 1)) dn$

$A = \frac{1}{2} \times 4 = 2$

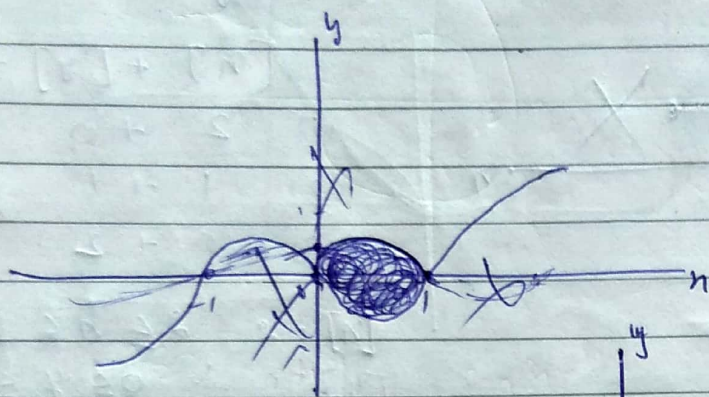
$A = \frac{1}{2} \times 4 = 2$

$$y^2 = 1 - n^2$$

$$y^2 + n^2 = 1$$

Ques: Find  $y = \sqrt{1 - n^2}$

$$y = n^3 - n = n(n^2 - 1) = n(n-1)(n+1)$$

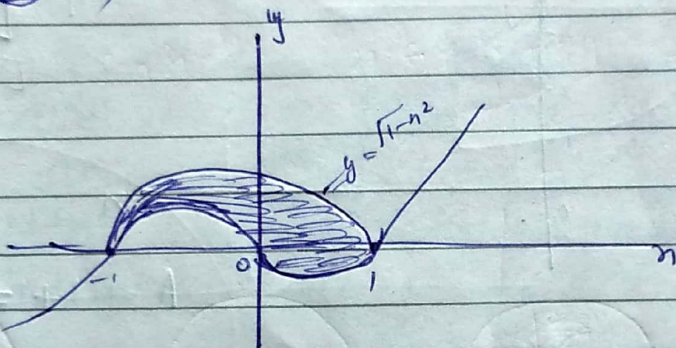


$$y = \sqrt{1 - n^2} \quad n=0, y=1$$

$$n=1, y=0$$

$$n=2, y = \sqrt{3}$$

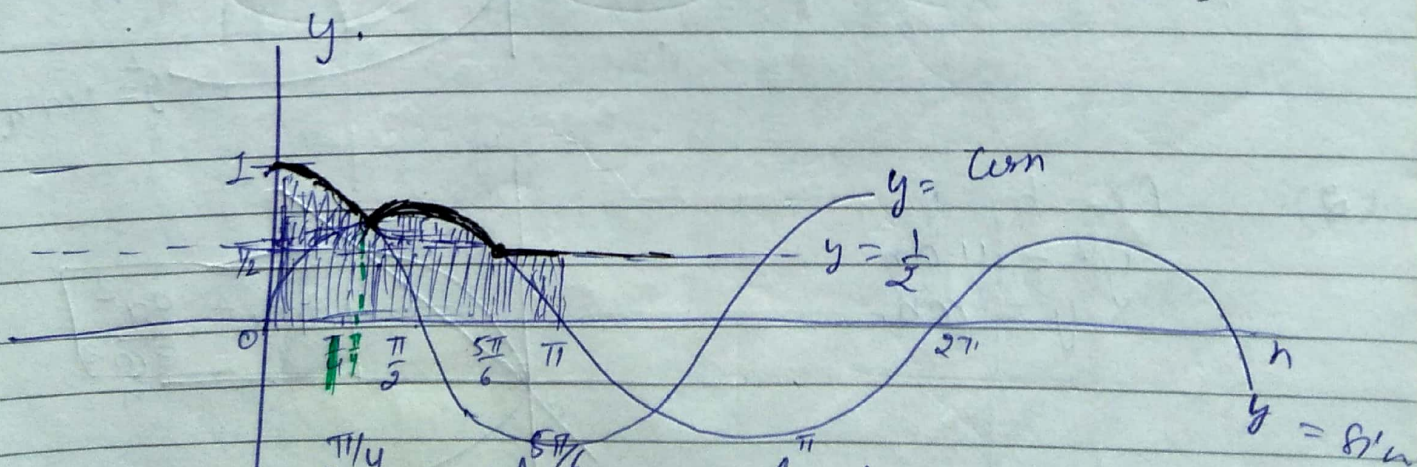
$$A = \frac{\pi}{2}$$



$$A = \frac{\pi}{2} = \frac{\pi(1)^2}{2} = \frac{\pi}{2}$$

Ques:  $f(n) = \max \left\{ \sin n, \cos n, \frac{1}{2} \right\}$

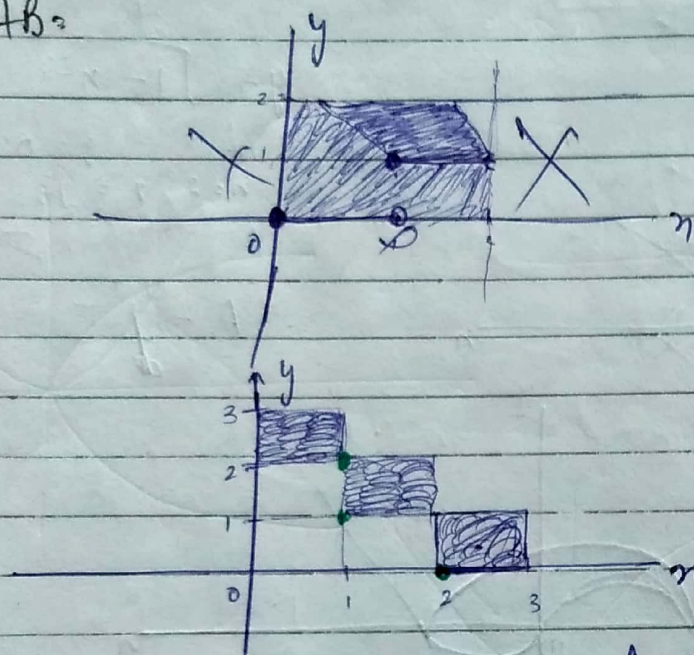
$$n=0, y=0, n \in (0, \pi)$$



$$A = \int_0^{\pi/4} \cos n + \int_{\pi/4}^{5\pi/6} \sin n + \int_{5\pi/6}^{\pi} \frac{1}{2} dn$$

Que!  $[x] + [y] = 2 \quad x, y \geq 0$

Ans:



$$x+y=2$$

$$[x] + [y] = 2$$

(1)  $2 + 0$

(2)  $1 + 1$

(3)  $0 + 2$

$$[x] = 2 \quad 2 < x < 3$$

$$[y] = 0 \quad 0 \leq y < 1$$

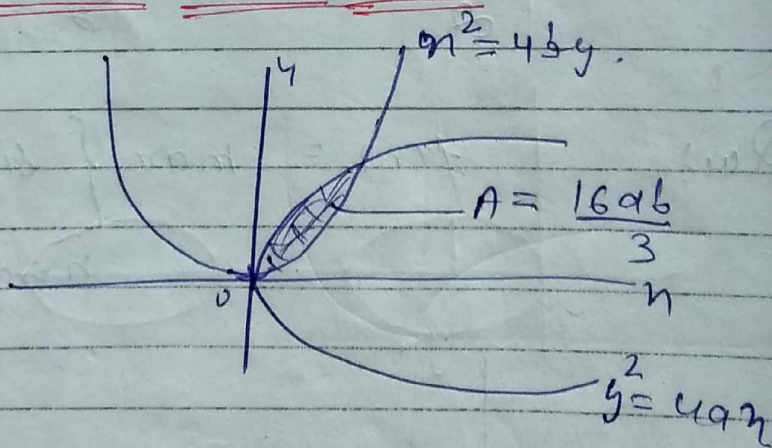
$$1 < x < 2, \quad 1 \leq y < 2$$

$$A = 1 + 1 + 0 = 2$$

Note! Standard Area to be Remembered.

(1)  $y^2 = 4ax$

$$x^2 = 4by$$

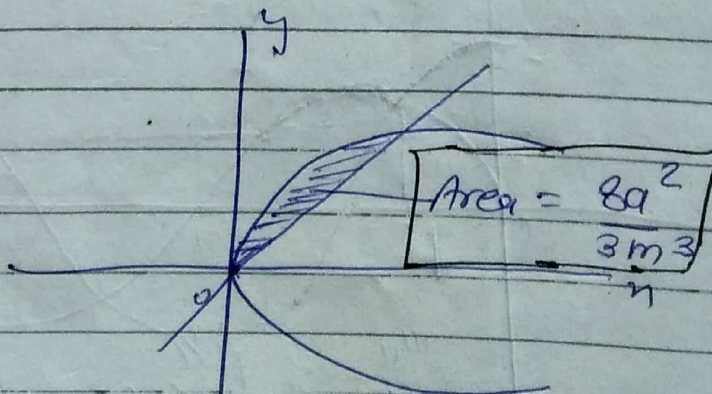


(2)

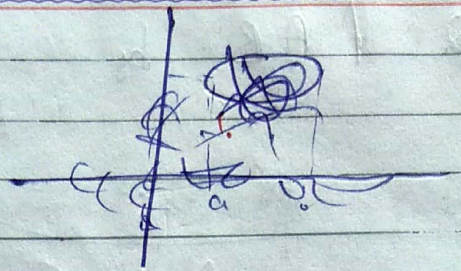
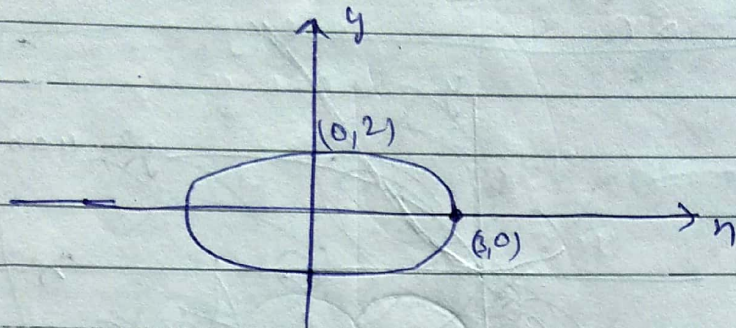
AB by

$$y^2 = 4ax$$

$$y = mx$$



(3)  $\frac{x^2}{a} + \frac{y^2}{b} = 1$



~~Area~~  $A = \pi ab$

~~Note~~ Note :-

~~(b)~~ Area Remains invariant even if the coordinate axes are shifted. Hence shifting of origin in many cases proves to be very convenient in computing Areas.

\*  $\max\{x, y\} = \frac{x+y}{2} + \left| \frac{x-y}{2} \right|$

$\min\{x, y\} = \frac{x+y}{2} - \left| \frac{x-y}{2} \right|$

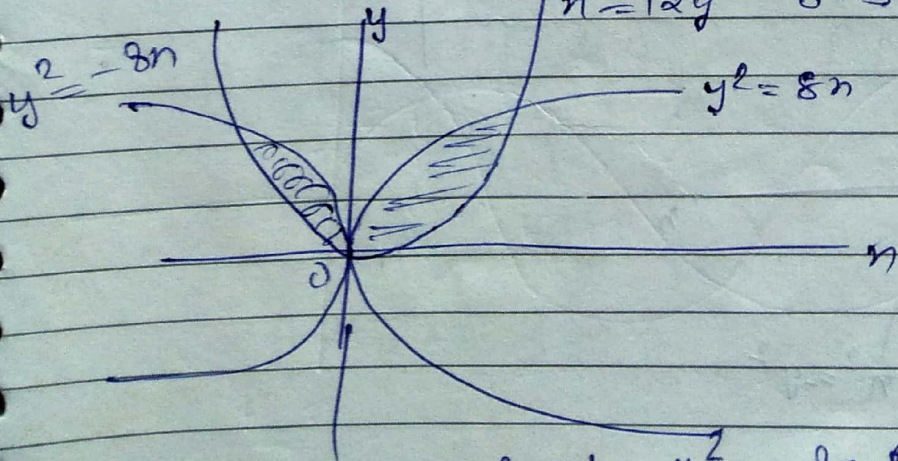
Ques Find AB

$y^2 = 8x$

$x^2 = 12y$

$x^2 = 12y$      $b=3$

$a=2$



$\frac{16ab}{3} = \frac{16 \cdot 2 \cdot 3}{3} = 32$

A.B by  $y^2 = -8x$  &  $x^2 = 12y \Rightarrow A = 32$

$n=1, y=1$   
 $n=2, y=4$   $n=-1$   
 $n=3, y=9$   $y=2$   
 $n=0, y=0$

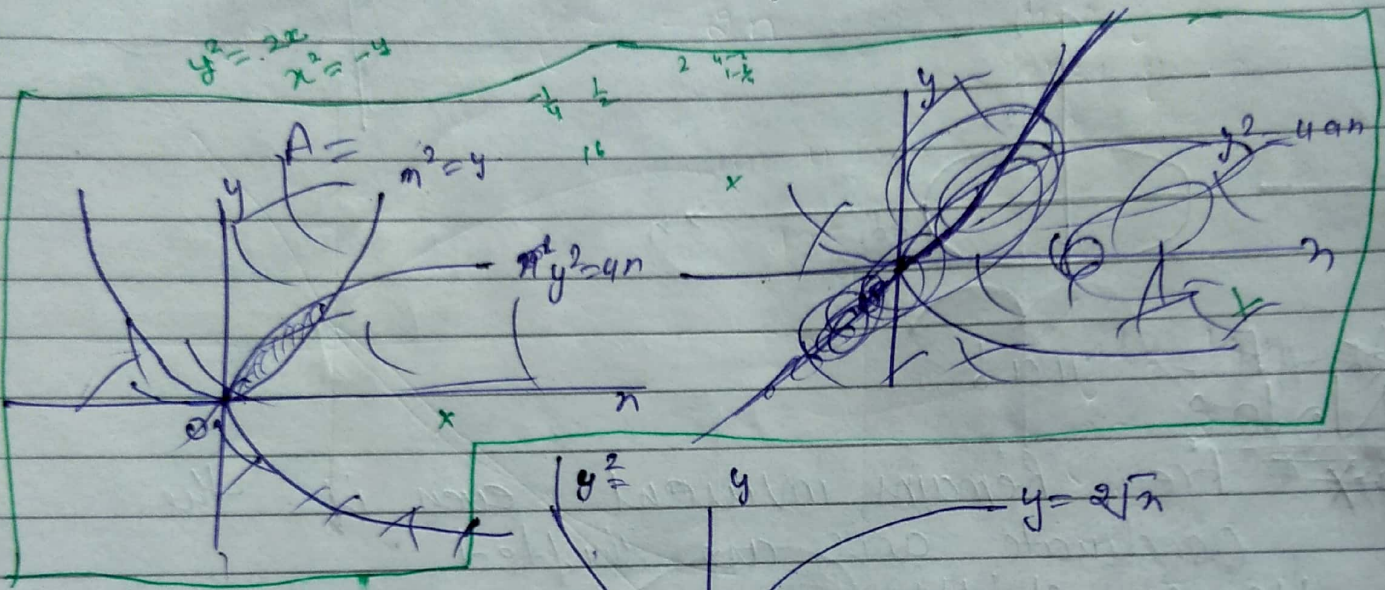
Q FAB by

$$y = 2\sqrt{n}$$

$$n = -\sqrt{y}$$

$$y^2 = 4n$$

$$n^2 = y$$

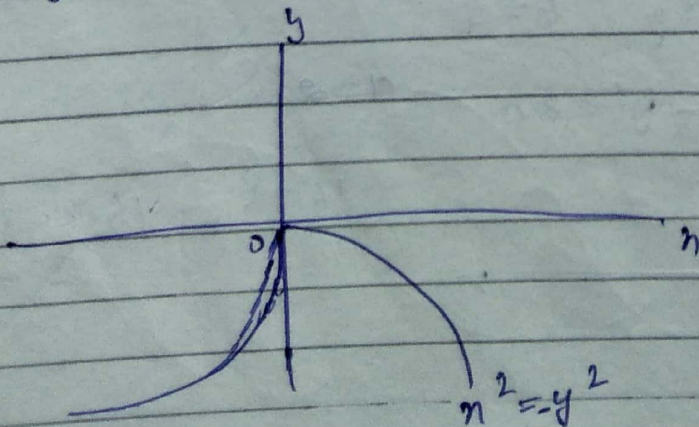
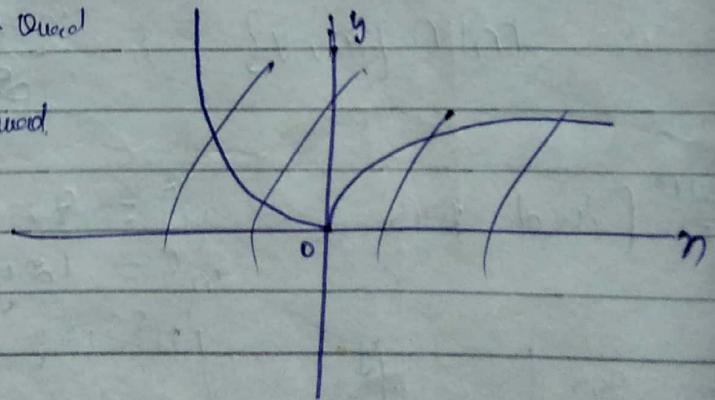


No Bounded region form.

Que!  $n = \sqrt{-y}$  → lie in 4th Quad  
 $y = \sqrt{-n}$  → lie in 3rd Quad

$$n^2 = -y^2$$

$$y^2 = -n$$



6. no. 5 → 1 page | 31 - 1  
 28 → 1 page | 39 - 1/2  
 30 → 1 page  
 34 → 1 page

Ques. FAB  $y^2 - 2y + 4n + 3 = 0$   
 $n^2 + 2n - y + 2 = 0$

$y^2 - 2y + 4n + 4 + 1 = 0$   
 $(y-1)^2 + 4(n+1) = 0$   
 $n^2 + 2n - y + 1 + 1 = 0$

$(n+1)^2 - (y-1) = 0$   
 $(n+1)^2 = (y-1)$   
 $(y-1)^2 = -4(n+1)$

$y-1 = Y \Rightarrow y=1$   
 $n+1 = X \Rightarrow n=-1$

$X^2 = Y \Rightarrow (n+1)^2 = (y-1)$   
 $Y^2 = -4X \Rightarrow (y-1)^2 = -4(n+1)$

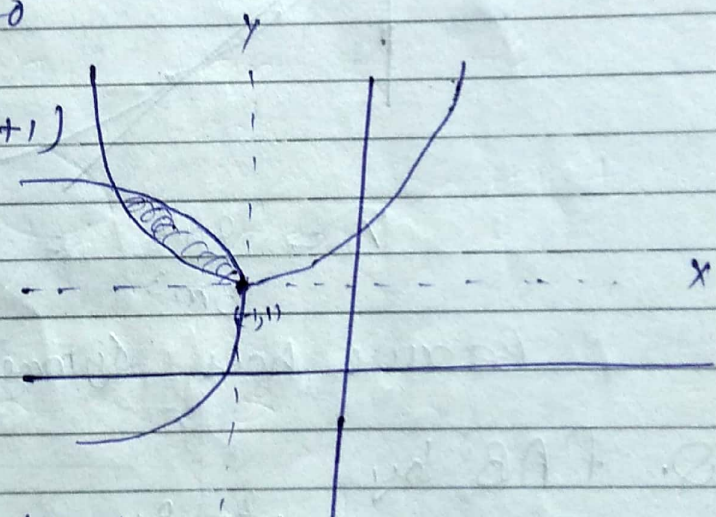
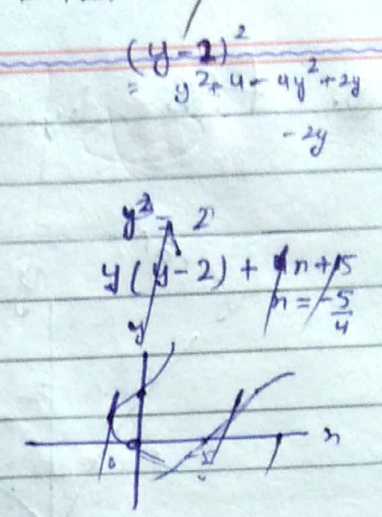
$a = \frac{1}{4} \quad b = 1$

$A = \frac{16ab}{3} \Rightarrow \frac{4}{3} A$

$Q = |n-1| + |y+3| \leq 4$  FAB

$n-1 = X$   
 $y+3 = Y$

Area =  $3a$



$|x| + |y| \leq 4$

$$\frac{8 \times 16}{3 \times 8} = \frac{16}{3}$$

Q. FAB by

$$y^2 = 16n$$

$$y = -2n$$

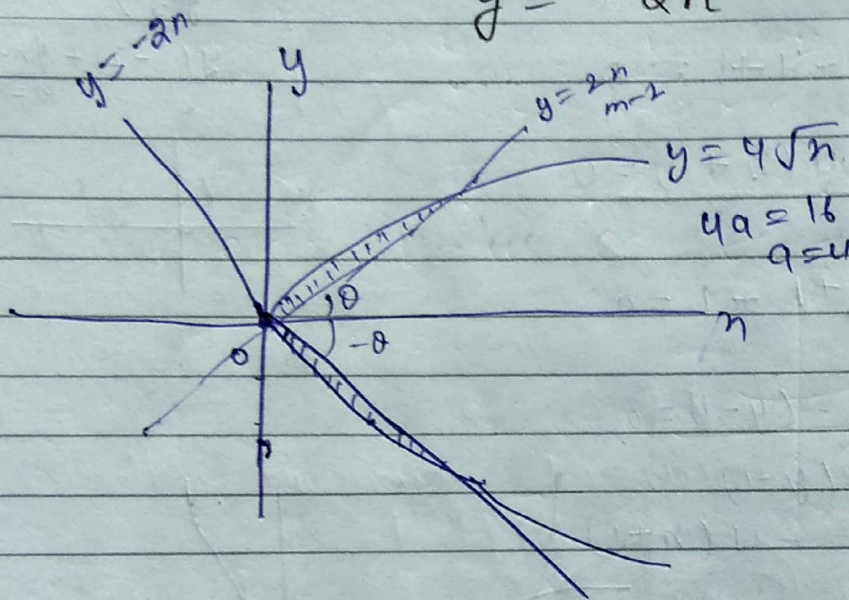
$$y = 4\sqrt{n}$$

$$y = -2n$$

$$n=0, y=0$$

$$n=1, y=-2$$

$$n=2, y=-4$$



$$4a = 16$$

$$a = 4$$

$$y^2 = 16n$$

$$y = 2n$$

Same as  
that

Area.

$$A = \frac{8a^2}{3m^3} = \frac{8 \cdot 16}{3 \cdot 8} = \frac{16}{3}$$

Because has symmetrical about n-axis.

Q. FAB by

$$n^2 = 8y$$

$$2y = n$$

$$y = \frac{n}{2}$$

$$2y = n$$

$$n=1, y=0.5$$

$$n=2, y=1$$

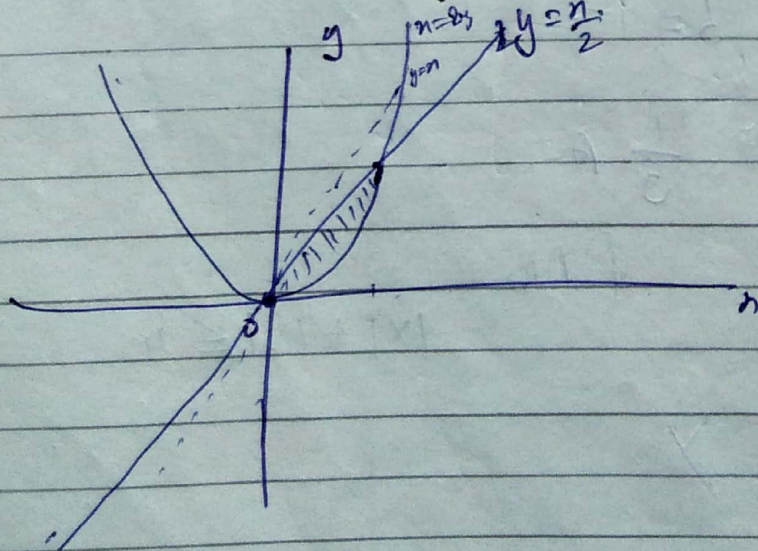
$$n^2 = 8y$$

$$y = \frac{n^2}{8}$$

$$n=1, y = \frac{1}{8} = 0.125$$

$$n=0, y=0$$

n=



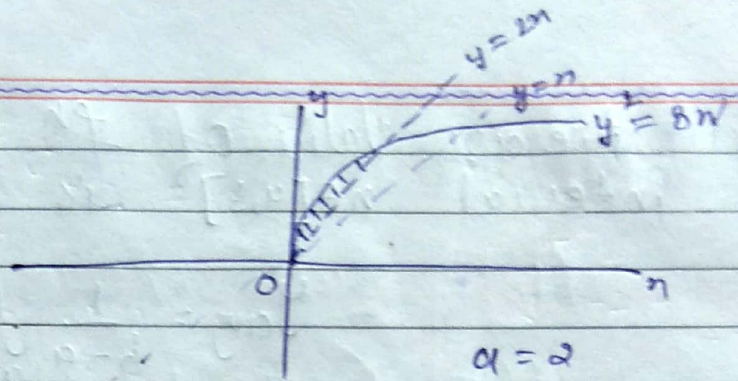


(No. 2)

Ans

$$y^2 = 8n$$

$$y = 2n$$



$$A = \frac{8 \cdot 4}{3 \cdot 8} = \frac{4}{3}$$

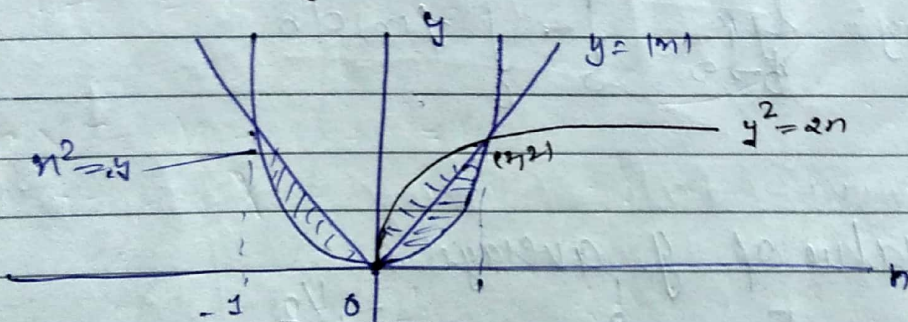
$$a = 2$$

$$m = 2$$

Note: Area bounded by  $n^2 = 8y$  &  $2y = n$  and AB by curve  $y^2 = 8n$  &  $y = 2n$  will be same.

Q.iii) FAB

$$n^2 = y \quad \& \quad y = |n|$$



$$m = 1, a = \frac{1}{2}$$

$$A = \frac{8a^3}{3m^3}$$

$$= \frac{8 \cdot \frac{1}{8}}{3} = \frac{2}{3}$$

$$A = \frac{4}{3}$$

$$= 2 \int_0^1 (n - n^2)$$

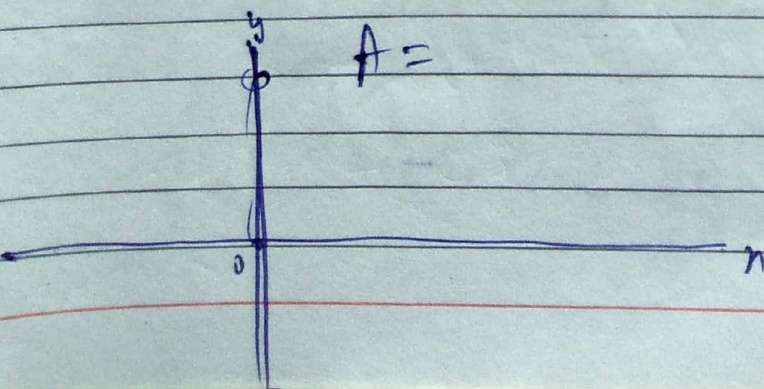
$$= 2 \times \left( \frac{n^2}{2} - \frac{n^3}{3} \right) \Big|_0^1$$

$$2 \times \left( \frac{1}{2} - \frac{1}{3} \right)$$

$$2 \times \frac{3-2}{6} = \frac{2 \times 1}{6} = \frac{1}{3} A$$

$$A = \frac{1}{3} A$$

Q. FAB =  $n^2 = 2y$  &  $y = |n|$

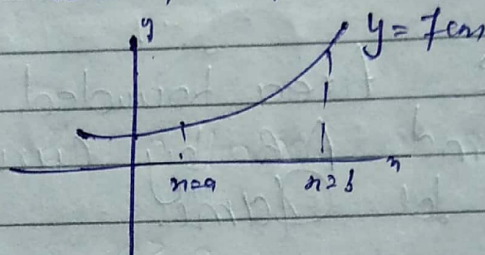


\* (i) Average value of  $f^n$   $y = f(x)$  in the interval  $x \in [a, b]$  is define as

$$Y_{avg} = \frac{1}{b-a} \int_a^b f(x) dx$$

• average value can be +ve, -ve, or zero.

(ii)



(ii) if  $f^4$  is define in the int.  $x \in (0, \infty)$

$$Y_{avg} = \lim_{b \rightarrow \infty} \frac{1}{b} \int_0^b f(x) dx$$

Root means square

(iii) RMS value of  $y$  average

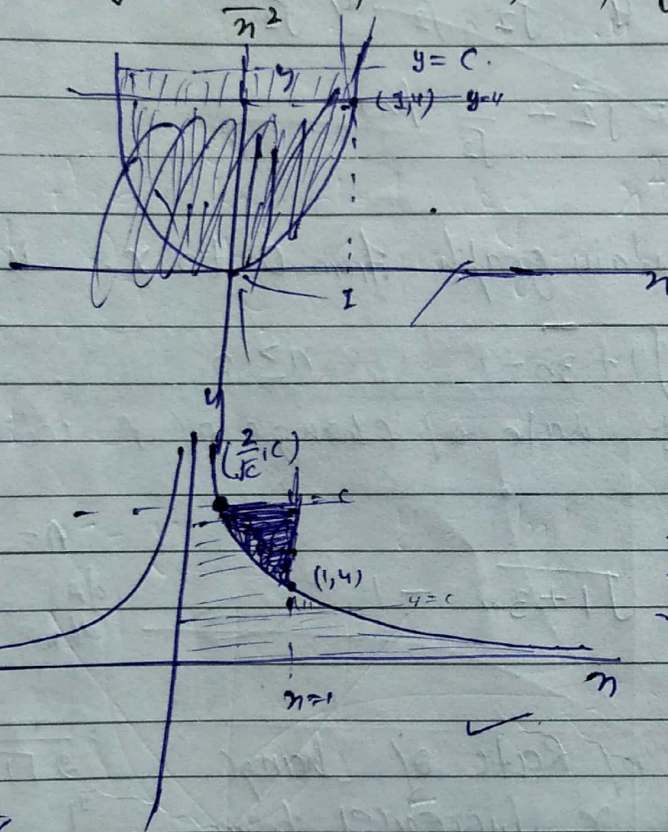
$$Y_{avg} = \left[ \frac{1}{b-a} \int_a^b f^2(x) dx \right]^{1/2}$$

# \* Determination of Parameter!

Ques Find value of  $C$  for which  $AB$  by Curve

$y = \frac{4}{n^2}$ ,  $n=1$ , &  $y=C$  is equal to  $\frac{9}{4}$

Ans



$$y = \frac{4}{n^2}$$

$$n=3, y=4$$

$$n=1, y=4$$

$A = \int_1^3 (4 - \frac{4}{n^2}) dn$

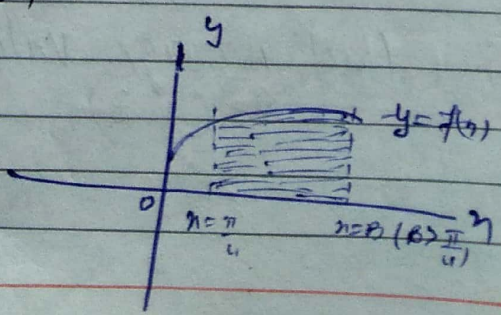
$$\int_{\frac{2}{\sqrt{C}}}^1 (C - \frac{4}{n^2}) dn = \frac{9}{4}$$

or

$$\frac{9}{4} = \int_{\frac{2}{\sqrt{C}}}^1 (\frac{4}{n^2} - C) dn$$

Ques: 2 let  $f(x)$  is non  $\phi$  -ve cont.  $f(x)$  such that  $AB$  by Curve  $y = f(x)$ ,  $x$  axis and the vertical  $x = \frac{\pi}{4}$  and  $x = \beta$  ( $\beta > \frac{\pi}{4}$ ) is equal to  $\beta \sin \beta + \frac{\pi}{4} \cos \beta + \sqrt{2} \beta$  then find  $f(\frac{\pi}{4})$

$$= \int_{\frac{\pi}{4}}^{\beta} f(x) dx = \beta \sin \beta + \frac{\pi}{4} \cos \beta + \sqrt{2} \beta$$



diff w.r.t  $\beta$ .

$$f(\beta) = \beta \cos \beta + \sin \beta - \frac{\pi}{4} \sin \beta - \frac{\pi}{4} \sin \beta + \sqrt{2}$$

$$f\left(\frac{\pi}{4}\right) = \frac{\pi}{4} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \frac{\pi}{4} \frac{1}{\sqrt{2}} + \sqrt{2}$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}} \text{ Ans}$$

Q. The AB by certain graph from 0 to  $x$ . is given by

$$A = \sqrt{1+3n} - 1, \quad n \geq 0$$

find instantaneous rate of change of  $A$  with  $n$  at  $n=5$ .

$$A = \int_0^n f(n) dn = \sqrt{1+3n} - 1$$

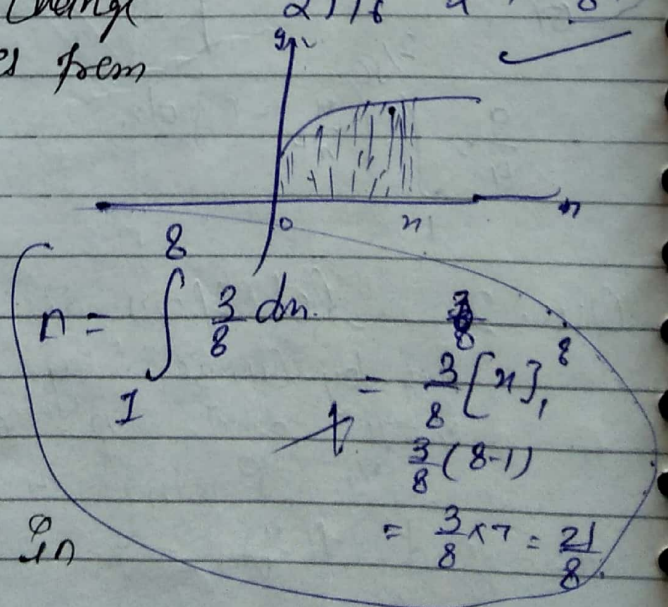
$$A = \sqrt{1+3n} - 1$$

$$\left(\frac{dA}{dn}\right)_{n=5} = \frac{3}{2\sqrt{1+3n}} \Big|_{n=5}$$

(ii) Find Average of Rate of Change w.r.t  $x$  as  $x$  increases from 1 to 8.

Average rate of change 1 to 8.

$$= \frac{\int_1^8 f(n) dn}{8-1}$$



$$n = \int_1^8 \frac{3}{8} dn$$

$$= \frac{3}{8} [n]_1^8$$

$$= \frac{3}{8} (8-1)$$

$$= \frac{3 \times 7}{8} = \frac{21}{8}$$

(iii) find Average value of  $f_n$  in  $n \in (1, 3)$

$$n^2 - 2n + 2$$

$$n^2 - 2n + 2$$

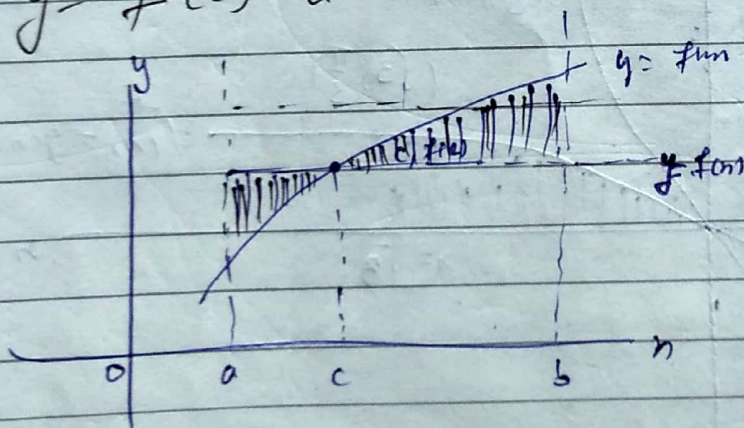
$$n^2 - (n+n) + 2$$

$$n(n-1) - 1(n-1) + 2$$

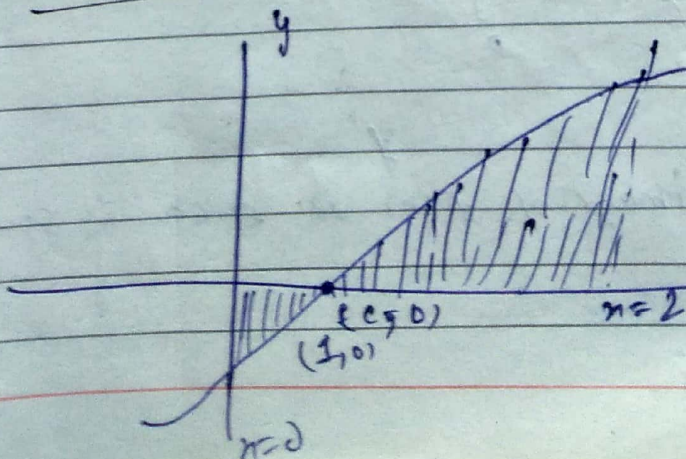
$$y_{avg} = \int_1^3 (n^2 - 2n + 2) dn$$

Note

Ques:  $y = f(x)$  is monotonic  $f'' > 0$  in  $(a, b)$   
 the AB by ordinates at  $x = a, x = b$   $y = f(x)$  and  
 $y = f(c)$  is minimum when  $c = \frac{a+b}{2}$   
 $c \in (a, b)$



Ques If AB by Curve  $f(x) = \frac{x^3}{3} - x^2 + 9$   
 and the lines  $x=0, x=2$ , and  $x$  axis is  
 minimum then find A



$$A = \int_0^2 (\frac{x^3}{3} - x^2 + 9) dx$$

$$= (\frac{x^4}{12} - \frac{x^3}{3} + 9x) \Big|_0^2$$

$$= \frac{16}{12} - \frac{8}{3} + 18$$

$$= \frac{4}{3} - \frac{8}{3} + 18$$

$$= \frac{-4}{3} + 18$$

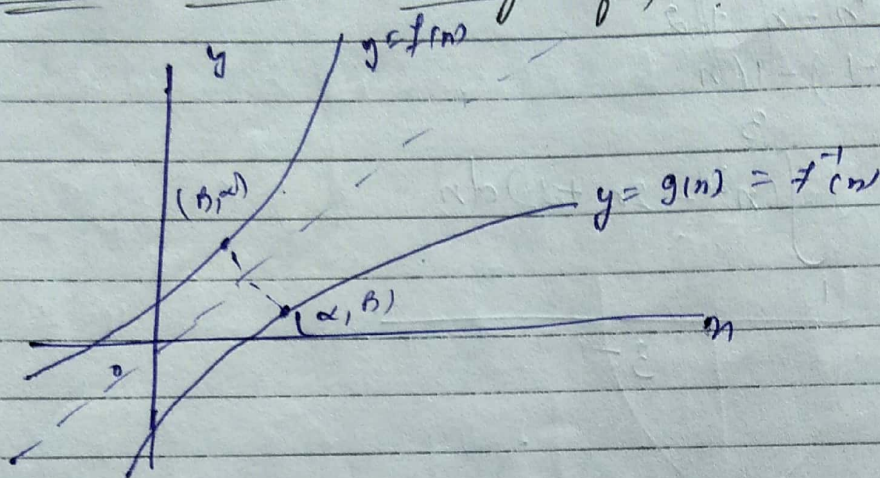
$$= \frac{-4 + 54}{3}$$

$$= \frac{50}{3}$$

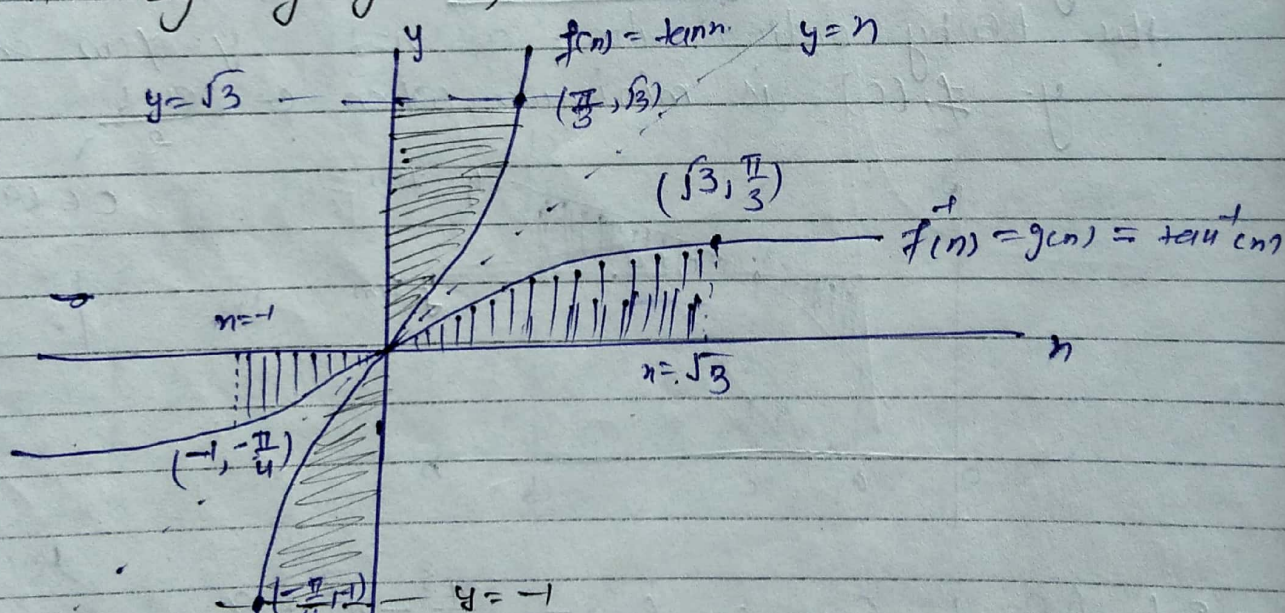
$$0 = \frac{1}{3} - 1 + 9$$

$$a = \frac{2}{3} \text{ Ans}$$

## \* Area Under the Image of $f^{-1}$



Que: Let  $f(x) = \tan x$  and  $g(x)$  is its inverse. Find Area  $A$  by  $y = g(x)$ ,  $x$  axis and ordinat  $x = -1$ , and  $x = \sqrt{3}$ .



$$A = \left| \int_{-1}^{\sqrt{3}} \tan^{-1} x \, dx \right| + \int_0^{\sqrt{3}} \tan^{-1} x \, dx = A_{\text{total}}$$

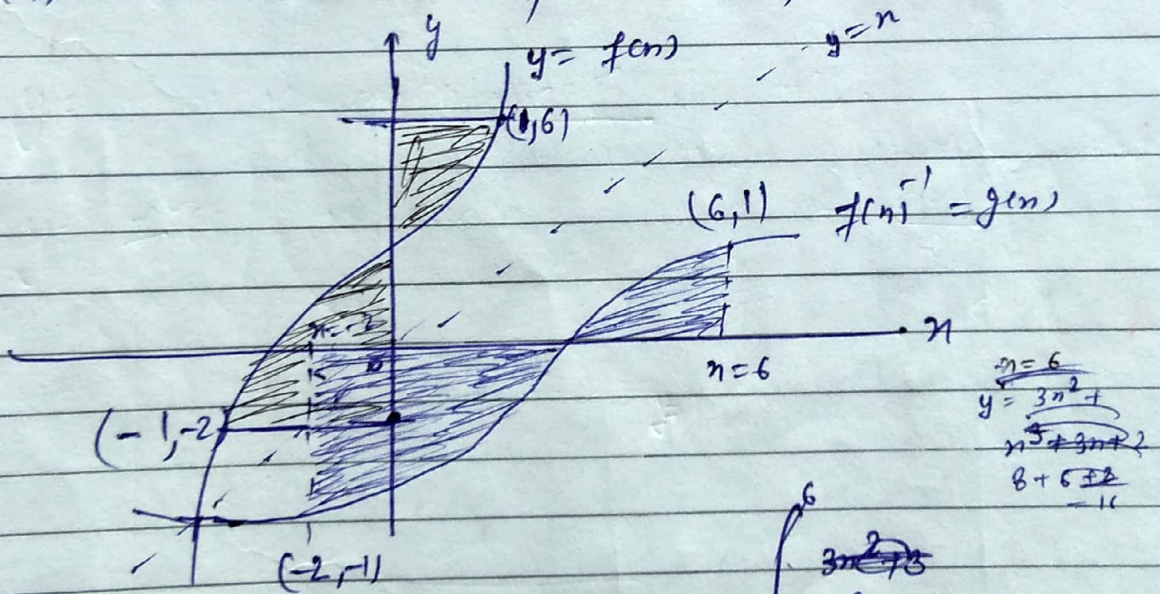
Method-2:

$f(x)$  mirror image of  $f^{-1}(x)$  is area is same of the  $f^{-1}(x)$

$$\int_0^{\pi/3} (\sqrt{3} - \tan n) dn + \int_{-\pi/4}^0 (\tan x - (-1)) dy = \dots$$

(ii) Let  $f(n) = n^3 + 3n + 2$  and  $g(m)$  is its inverse.  
 Find Area by  $y = g(m)$ ,  $n$  axis and the ordinate  
 at  $n = -2$ , at  $n = 6$ .

$$f'(n) = 3n^2 + 3 > 0 \Rightarrow f \text{ is m.f. } f^4.$$



$$\int_{-1}^1 n^3 + 3n + 2$$

$$A = \int_0^1 (6 - f(n)) dn + \int_{-1}^0 (f(n) - (-2)) dn$$

$$\int_{-2}^6 (3n^2 + 3n + 2) =$$