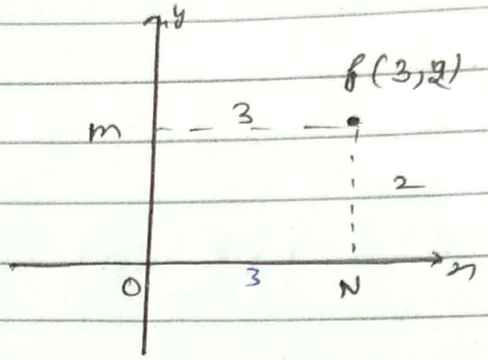
SBG STUDY

12/09/17

$\perp$ cular
if x y z, x cordinate in yz Plane.
y " " in xz Plane
z " " xy Plane

* 3 D *

* eqn of xy Plane $z=0$
  yz " $x=0$
  x~~z~~ " $y=0$



Q. Find locus of point whose x coordinate is 3
Ans: it is Plane parallel to yz Plane and at a dist. 3 unit from it.

Q.2 Find locus of a point whose y coordinate is 3 and x cordinate is 2.
Ans: Line obtained by intersection of two Planes one is Parallel to yz plane ~~and~~ another is Parallel to xz plone

Q.3 Find locus of a point whose $x=3, y=2, z=4$
(~~3,0,0~~) (0, 2, 0) . (0, 0, 4)

* $P(\alpha, \beta, \gamma)$ → ⊥ dist from P to xy plane
  β ↓ ⊥ dist from P to xz Plan
  α ↓ ⊥ dist from P to Yz Plane

Direction Cosines (DC)

if any line makes an angle (α, β, γ) with Positive x, y, z respectively then these are called their dirn angle and Cosine to these angle are called dirn Cosine.

α, β, γ are dirn angle of line

$\cos\alpha = l$, $\cos\beta = m$, $\cos\gamma = n$ are dirn Cosine (dc) of given line

* $-l, -m, -n$ are represent dirn cosine of same lin

Direction Ratio (DR)

if $\bar{a}, \bar{b}, \bar{c}$ are three no. and which are proportional to direction cosine are called Direction Ratio of the given line

$\Rightarrow \langle a, b, c \rangle$

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$$

$\langle l, m, n \rangle$ are dc.

ratio formula $= \pm \dfrac{\sqrt{l^2+m^2+n^2}}{\sqrt{a^2+b^2+c^2}}$

$l^2+m^2+n^2 = 1$ or
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

dR $\propto$ dC

i j + k के coff. DR है $\cos\alpha = \frac{b}{H}$.

$\therefore\ l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}}$; $m = \pm \frac{b}{\sqrt{a^2+b^2+c^2}}$;

$n = \pm \frac{c}{\sqrt{a^2+b^2+c^2}}$

* if $A(x_1, y_1, z_1)$ & $B(x_2, y_2, z_2)$ are two Points then DR of lin AB is

$$\boxed{\langle \underbrace{x_2 - x_1}_{a}, \underbrace{y_2 - y_1}_{b}, \underbrace{z_2 - z_1}_{c} \rangle}$$

$A(x_1, y_1, z_1)$ —— $B(x_2, y_2, z_2)$

* $\boxed{AB = \underbrace{(x_2 - x_1)}_{DR}\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}}$ Imp.

$\vec{OP} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$.

$|\vec{OP}| = \sqrt{a_1^2 + a_2^2 + a_3^2} = r$

$\cos\alpha = \frac{a_1}{r}$

$\cos\beta = \frac{a_2}{r}$

$\cos\gamma = \frac{a_3}{r}$

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{a_1^2 + a_2^2 + a_3^2}{a_1^2 + a_2^2 + a_3^2} = 1}$$

vw!

DC unit vector.   DR = l m n α.

$$\hat{OP} = \frac{\vec{OP}}{|\vec{OP}|} = \frac{a_1}{\sqrt{a_1^2+a_2^2+a_3^2}}\hat{i} + \frac{a_2}{\sqrt{a_1^2+a_2^2+a_3^2}}\hat{j} + \frac{a_3}{\sqrt{a_1^2+a_2^2+a_3^2}}$$

$= l\hat{i} + m\hat{j} + n\hat{k}$ ∧ – di C.

Dir^n Cosines

DC

* if unit vector write then it is DC.
if unit vector not write down then it is DR.

Ex $\vec{OQ} = 2\hat{i} + \hat{j} + 2\hat{k}$. $\hat{OQ} = \frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$

dr of line OQ is ⟨2, 1, 2⟩

dr " " " is ⟨−4, −2, −4⟩

dr " " " " ⟨6, 3, 6⟩

dC of line OQ is $\langle \frac{2}{3}, \frac{1}{3}, \frac{2}{3} \rangle$

dr of lin OQ is ⟨2, 1, 2⟩

$$l = \frac{2}{\sqrt{2^2+1^2+2^2}} = \frac{2}{3}$$

$OP = a_1 i + a_2 j + a_3 k$

$OQ = b_1 i + b_2 j + b_3 k$

$$\cos\theta = \frac{\overline{OP} \cdot \overline{OQ}}{|\overline{OP}|\,|\overline{OQ}|} = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}}$$

dr of OP line $\langle a_1, a_2, a_3 \rangle$ or dc is $\langle l_1, m_1, n_1 \rangle$

" " OQ = " $\langle b_1, b_2, b_3 \rangle$ or dc is $\langle l_2, m_2, n_2 \rangle$

or $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$.

* if lines are Parallel then

$$\boxed{\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}}$$ or $l_1 = l_2$, $m_1 = m_2$, $n_1 = n_2$

(ii) If lines are ⊥ular then

~~$a_1 a_2 + b_1 b_2 + b$~~

$$a_1 b_1 + a_2 b_2 + a_3 b_3 = 0$$

or

$$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$$

$\frac{180}{3} = 60$

Q: find locus of point whose l is $= \frac{1}{2}$.

$= \quad \cos\alpha = \frac{1}{2} \quad \alpha = \frac{\pi}{3}$

* $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2.$

* DC of vector is $2i - j - 2k$ is

$\cancel{2i - j - 2k}$

$= \frac{2}{3}i - \frac{1}{3}j - \frac{2}{3}k \quad$ i.e $dc = \frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$

$dc = \left(-\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$

$dr = (2, -1, -2)$

$dr \langle 4, -2, -4 \rangle$

Que: find dc of line which is equally inclind with all three axis.

$l^2 + m^2 + n^2 = 1$

$= l = m = n$

$\cos^2\alpha + \cos^2\beta$

$\cos^2\alpha = \cos^2\beta = \cos^2\gamma$

$3l^2 = 1$

$l = \pm\frac{1}{\sqrt{3}}$

$m = n = l = \pm\frac{1}{\sqrt{3}}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma =$

$= \frac{\ell}{\ell} + \underline{m}$

$= \cos^2\alpha + \cos^2\beta + \cos^2\gamma$

$= \frac{l^2 + m^2 + n^2}{\sqrt{a^2+b^2+c^2}}$

$a^2 + \quad = dk$

$a' \quad = dA$

(iv) Ans - no. of such vector are 8 $\qquad \lambda = \pm\frac{1}{\sqrt{3}}$

line = 4.

dr of line $\qquad$ vector = 8.

$$\begin{array}{ccc}
\lambda & \lambda & \lambda \\
\lambda & \lambda & -\lambda \\
\lambda & -\lambda & \lambda \\
+\lambda & -\lambda & -\lambda \\
-\lambda & \lambda & \lambda \\
-\lambda & \lambda & -\lambda \\
-\lambda & -\lambda & \lambda \\
-\lambda & -\lambda & -\lambda
\end{array}$$

where $\lambda = \frac{1}{\sqrt{3}}$

Note:

Que: find dr of the line $\perp$cular to two lines

whose dc are $\langle 1, 2, 3 \rangle$

& $\langle -2, 1, 4 \rangle$

shortest dist.

Suppose dr of line is $\langle a, b, c \rangle$

$= \frac{a}{\sqrt{14}}, \frac{b}{\sqrt{14}}, \frac{c}{\sqrt{14}}$

$a + 2b + 3c = 0$

$-2a + b + 4c = 0$

$$\begin{array}{ccc}
1 & 2 & 3 \\
-2 & 1 & 4
\end{array}$$

Solve by ~~Staben~~ Rule

Solve by Cramer Rule

IX

$$\frac{a}{\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}} = \frac{b}{\begin{vmatrix} 1 & 3 \\ -2 & 4 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix}}$$

$$= \frac{a}{5} = \frac{b}{-10} = \frac{c}{5}$$

$\therefore$ dr of Required line is $(5, -10, 5)$

or $(1, -2, 1)$

M-2

dr $\perp^r$ to both line

$$= \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ -2 & 1 & 4 \end{vmatrix} = 5i - 10j + 5k.$$

Que: dc of two lines are connected by the relation

$$l + m + n = 0$$

&

$$2lm + 2ln - mn = 0$$

then find angle b/w them.

$$l = -(m+n)$$

$$2l(m+n) - mn = 0$$

$$2(m+n)^2 + mn = 0$$

$$2m^2 + 5mn + 2n^2 = 0$$

$$2m^2 + 4mn + mn + 2n^2 = 0$$

$$2m(m+2n) + n(m+2n) = 0$$

$$(2m+n)(m+2n) = 0$$

Square sum = DC.

for Line 1

for Line 2

$2m + n = 0$

$l + m + n = 0$

$l + m + (-2m) = 0$

$l = m$

$\therefore (m : m : -2m)$

or $(1 ; 1, -2$

$m + 2n = 0$

$l + n + n = 0$

$l + (-2n) + n = 0$

$l = n$

$\therefore \langle n, -2n, n \rangle$

or $(1, -2, 1)$

$$\cos\theta = \frac{1.1 + 1(-2) + (-2)(1)}{\sqrt{1^2 + 1^2 + (-2)^2}\sqrt{1^2 + (-2)^2 + 1}}$$

Equation of plane :

Plane is surface such that line segment joining any two point on the surface lies completely on it.

General eq of plane in cartisian form is

$$Ax + By + Cz + D = 0$$

where Dr normal to the plane is $\langle A\ B\ C \rangle$ or

normal vector to the plane is

~~$\langle A\hat{i} + B$~~ $\boxed{\vec{n} = A\hat{i} + B\hat{j} + C\hat{k}}$

a, b, c

* eq^n of Plane in Various forms:

$$\frac{A}{D}x + \frac{B}{D}y + \frac{C}{D}z + 1 = 0$$

* Three information is needed to write a plane unikely.

(1) A point and vector normal to it is given.

$A(x, y, z)$ or $A(\bar{a})$

$\bar{a} = x_1 i + y_1 j + z_1 k.$

$P(r)$ or $P(x, y, z)$

$\overline{PA} \perp$ $\quad \overline{PA} \cdot \bar{n} = 0$

$$\boxed{(\bar{r} - \bar{a}) \cdot \bar{n} = 0}$$

$r \cdot n = \bar{a} \cdot \bar{n}$ $\quad \boxed{r \cdot n = \text{Constant}}.$

* Cartisian form

$$Ax + By + Cz + D = 0 \quad \text{—} (1)$$

$$Ax_1 + By_1 + Cz_1 + D = 0 \quad \text{—} (2)$$

$$\boxed{A(x - x_1) + B(y - y_1) + C(z - z_1) = 0}$$

* Plane passes through $(1, -2, 3)$

Eq^n of plane

$$(x-1)A + (y+2)B + (z-3)C = 0$$

* Plane passes through 3 Points

$$A(x_1, y_1, z_1) \quad B(x_2, y_2, z_2) \quad C(x_3, y_3, z_3)$$

$$(x-x_1)A + (y-y_1)B + (z-z_1)C = 0$$

$$(x-x_2)A + (y-y_2)B + (z-z_2)C = 0$$

$$(x-x_3)A + (y-y_3)B + (z-z_3)C = 0$$

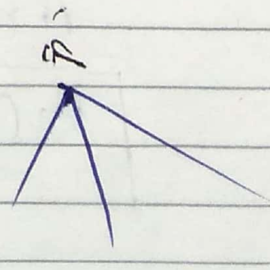
Now eliminate A, B, C and we will get eq of plane.

Hence eq^n of plane is

Cartesian form

$$\boxed{\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x-x_2 & y-y_2 & z-z_2 \\ x-x_3 & y-y_3 & z-z_3 \end{vmatrix} = 0}$$

Vectorical form : $[\overline{PA} \quad \overline{PB} \quad \overline{PC}]$



* Normal form of eqⁿ of plane:

$$An + By + Cz = D$$

$$\frac{\bar{r}\cdot\bar{n}}{|\bar{n}|} = \frac{\text{Const}}{|\bar{n}|}$$

$$\boxed{\bar{r}\cdot\hat{n} = d}$$ eqⁿ of Normal plane

Here, $\hat{n}$ is unit vector

General form of plane $\Rightarrow an + by + cz + d = 0$

$$\hat{n} = \frac{a}{\sqrt{a^2+b^2+c^2}}i + \frac{b}{\sqrt{a^2+b^2+c^2}}j + \frac{c}{\sqrt{a^2+b^2+c^2}}k$$

Normal form of plane $\Rightarrow ln + my + nz + \frac{d}{\sqrt{a^2+b^2+c^2}} = 0$

$$ln + my + nz = p \qquad p = \frac{-d}{\sqrt{a^2+b^2+c^2}}$$

Where (+)ve value of P, in normal form of eqⁿ of plane, gives ⊥ular dist. from origin to the plane

General form of plane $\Rightarrow 2n - y + 2z = 27$ $\qquad \sqrt{2^2+(-1)^2+2^2} = 3$

Normal form of plane:

$$\frac{2}{3}i - \frac{1}{3}y + \frac{2}{3}z = \frac{27}{3}$$

$= 9$

Wt whose Constant. $2x+3y=c$ $\frac{x}{3}+$.

** Here $\perp$cular dist. from Origin to the given Plane is q.

* **Intercept form of eq^n of plane :**

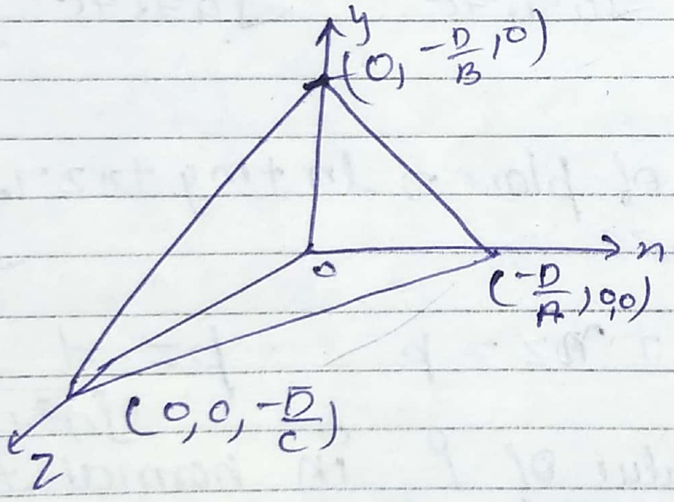
$$Ax + By + cz + D = 0$$

$$\frac{x}{-D/A} + \frac{y}{-D/B} + \frac{z}{-D/C} = 1$$

intercept made by plane upon x-axis is $= -D/A$

" " y-axis " $= -D/B$

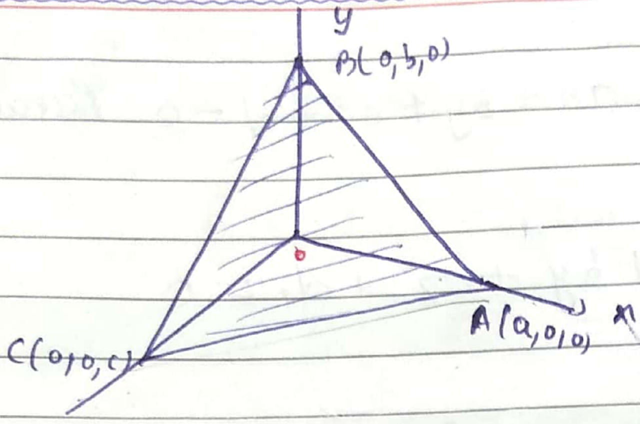
" " z-axis " $= -D/C$



i.e if a plane makes an intercept of length a, b, c respectively upon x, y, z respectively

when it eq is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$



eq^n of plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

* Area of $\Delta ABC = \frac{1}{2}|\bar{AB} \times \bar{AC}|$

$= \frac{1}{2}(-a\hat{i} + b\hat{j}) \times (-a\hat{i} + c\hat{k})$

$= \frac{1}{2}|bc\hat{i} + ca\hat{j} + ab\hat{k}|$

$\boxed{= \frac{1}{2}\sqrt{a^2b^2 + b^2c^2 + c^2a^2}}$

$= \sqrt{A_x^2 + A_y^2 + A_z^2}$

$A_x =$ Area of $\Delta OBC = \frac{1}{2}bc$

$A_y = \frac{1}{2}ac$

$A_z = \frac{1}{2}ab$

* In above fig. same times OABC is treated as tetrahedron the its vol. is given by $V = \frac{1}{6}[\bar{OA}\ \bar{OB}\ \bar{OC}]$

$= \frac{1}{6}\begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix} = \frac{abc}{6}$

** 1)

eqⁿ of plane $ax + by + cz + d = 0$ parallel to the given plane is

$P_2(x_0, y_0, z_0)$ or $P_2 = ax + by + cz + d_1 = 0$

(2) Dist. b/w two ||el Plane =

$$= \left| \frac{d_1 - d_2}{\sqrt{a^2 + b^2 + c^2}} \right|$$

** $A(x_1, y_1, z_1)$

P_1 or $P(x\ y\ z) = ax + by + cz + d = 0$

⊥ cular dist from Point $A(x_1, y_1, z_1)$ to the given Plane, $P(x, y, z) = ax + by + cz + d = 0$

$$\text{dist} = \left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a_1^2 + b^2 + c^2}} \right|$$

** P (or $P(x, y, z) = a_1 x + b_1 y + c_1 z + d_1 = 0$

$a_2 x + b_2 y + c_2 z + d_2 = 0$

Plane are ||el

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \neq \frac{d_1}{d_2}$

Identical

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{d_1}{d_2}$

⊥ cular

$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

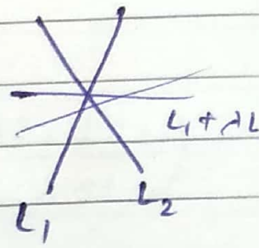
*

P or $P(x, y, z) = a_1x + b_1y + c_1z = 0$

$P_2(x, y, z) = a_2x + b_2y + c_2z = 0$

Angle b/w Plane P_1 and P_2 = angle b/w their Normal

$$\cos\theta = \left| \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$



Plane passes throw Line of Intersection of both Planes is given by $P_1 + \lambda P_2 = 0$

Normal to first plane $\vec{n_1} = a_1i + b_1j + c_1k$

second = $\vec{n_2} = a_2i + b_2j + c_2k$.

Dir^n of Line of Intersection of plane is

$\bar{n_1} \times \bar{n_2}$

* B·O Angle b/w the Planes

$P(x, y, z) = (a_1x + b_1y + c_1z + d_1 = 0$

$P_2(x, y, z) = a_2x + b_2y + c_2z + d_2 = 0$

eq^n of B·O angle is given by $= a_1x + b_1y + c_1z$

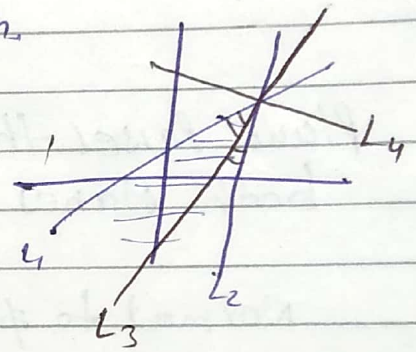
first of all Here Const. term of both plane is same sign.

eqⁿ of BOA $\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}} = \pm \frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}$

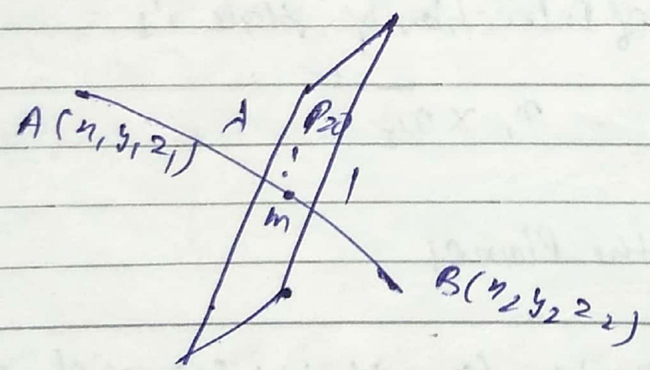
(i) +ve sign gives BOA Contain of origin and −ve BOA doesnot Contain origin

(ii) if $a_1a_2+b_1b_2+c_1c_2>0 \Rightarrow$ origin Lies in the obtuse angle Reigen.

other wise aclute angle Reigen.



* $P(x,y,z) = ax+by+cz+d=0$



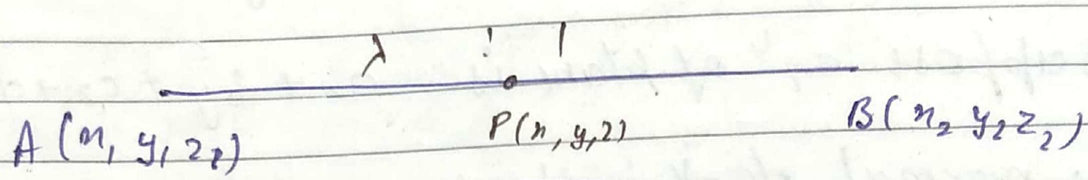
Line joining A & B will devide in ratio of $\lambda:1$ where,

$$\lambda = \frac{AM}{BM} = -\frac{P(x_1y_1z_1)}{P(x_2y_2z_2)}$$

$\lambda = -ve$ External

* if $ax_1 + by_1 + cz_1 + d$ and $ax_2 + by_2 + cz_2 + d$ have same/opp. sign then points $A(x_1 y_1 z_1)$ and $B(x_2 y_2 z_2)$ lie on the same/opp. sign.

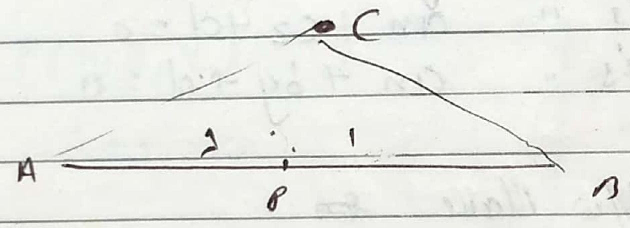
**



* $AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_2 - z_1)^2}$

* if Point P divide line joining AB in the Ratio of $\lambda : 1$ then

$$(x, y, z) \equiv \left(\frac{\lambda x_2 + x_1}{\lambda + 1}, \frac{\lambda y_2 + y_1}{\lambda + 1}, \frac{\lambda z_2 + z_1}{\lambda + 1} \right)$$



$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right)$$

** (1) eq^n of xy Plane is $z = 0$
(ii) " " xz " " $y = 0$
(iii) " " y~~x~~ " " ~~x~~$= 0$

(2) (i) eq^n of Plane ll to xy plane and at a dist d from it is $z = d$
(ii) " " " " xz Plane " " " is $y = d$
(iii) " " " yz " " $x = d$

(3) eq^n of plane $\parallel$el to x axis is

i.e normal to the plane is always $\perp^{cular}$ to x axis.

$\Rightarrow$ Suppose eq^n of plane is $ax + by + cz + d = 0$

$\therefore$ normal to this plane is $\bar{n} = a\hat{i} + b\hat{j} + c\hat{k}$

& vector along x-axis is $= \bar{n}_2 = \hat{i}$

$$\bar{n} \cdot \bar{n}_2 = 0$$

$$a = 0$$

$$by + cz + d = 0$$

Parallel to x-axis is $by + cz + d = 0$

" " y-axis " $ax + cz + d = 0$

" " z-axis " $ax + by + d = 0$

Que! find dist. b/w plane

$$2x - 3y + z = 4 \qquad 2x - 3y + z - 4 = 0$$

$$4x - 6y + 2z + 7 = 0$$

$$(2x - 3y + z + \tfrac{7}{2}) = 0$$

Parallel planes.

$$D = \left| \frac{-4 - \frac{7}{2}}{\sqrt{4+9+1}} \right|$$

$$\frac{2}{4} = \frac{-3}{6} = \frac{1}{2} \neq \frac{-4}{7} \quad \text{Not Identical.}$$

NO H.W.

Q. $2x - 3y + z - 4 = 0$
write eqⁿ of plane in Normal form.

$\sqrt{4+9+1} = \sqrt{14}$

$$\frac{2}{\sqrt{14}}x - \frac{3}{\sqrt{14}}y + \frac{1}{\sqrt{14}}z = \frac{4}{\sqrt{14}}$$

Q. find ⊥ dist from origin to above plane

$$= \frac{4}{\sqrt{14}}$$

Que. Let $P(x, y, z)$ be any point in the space.
find dist. of Point P from x axis.

Ans.
$P(x, y, z)$

~~$x\hat{i} + y\hat{j} + z\hat{k}$~~

$= d.$

$OP = \sqrt{y^2 + z^2}$

$y = 0,\ z = 0$

$(x, y, z) \quad (x, 0, 0)$

$= P(x, y, z) = 0$

$(x - d)\hat{i} + (y\hat{j} + z\hat{i}) = 0$

Que. find eqⁿ of plane passes throw $(2, -3, 1)$
& Parallel to $2x - y - z = 5$

$2x - 3y + z = 0$

$2x - y - z = 5$

$=$

$2x - y - z = \lambda$

$2 \cdot 2 + 3 - 1 = \lambda$

$\lambda = 6$

Q. find eq^n of plane $\parallel^{to}$ $x-2y+2z=5$ and at a dist 2 unit from it

Ans,

$$x-2y+2z=5$$

$2x+4$

$$\therefore x-2y+2z=\lambda$$

$$\frac{5-2}{\sqrt{1+4+4}}$$

$$\left|\frac{5-\lambda}{\sqrt{1^2+(-2)^2+2^2}}\right|=2$$

$$|\lambda-5|=6$$

$$\lambda=5\pm6$$

$$\lambda=11$$

$$\lambda=-1$$

Q. find eq^n of plane passes throw $(1,0,-2)$ & $\perp^r$ to $2x-y-z=2$ & $x-y-z=3$

$$2x+y-z-2=0$$

$$x-y-z-3=0$$

$$\frac{2+0+2-2}{\sqrt{4+1+1}}=$$

$=-2$

$$\frac{1+0+2+3}{\sqrt{1+1+1}}$$

$=$

Ans: Required Normal to the Plane = $\begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & -1 & -1 \end{vmatrix}$

$= -2i + j - 3k$

eq^n of plane :
$(x-1)(-2) + (y-0)(1) + (z+2)(-3) = 0$

M-2 $(x-1)A + (y-0)B + (z+2)C = 0$

$2A + B - C = 0$
$A - B - C = 0$

A, B, C $\begin{vmatrix} x-1 & y-0 & z+2 \\ 2 & 1 & -1 \\ 1 & -1 & -1 \end{vmatrix} = 0$

Que: Find eq^n of plane passes thro A $(1, 2, -1)$
& B$(0, 1, -2)$
& ⊥ ar to plane $x - 2y + 2z = 3$

Ans! $\cancel{i + 2j - k}$
$\cancel{j + 2k}$
$\cancel{A \times B}$
$\cancel{-3i + 2j + k}$.

$\cancel{\begin{vmatrix} i & j & k \\ 0 & 2 & - \\ 0 & 1 & 2 \end{vmatrix}}$

$\cancel{= -3i + 2j + k}$

x

$(x-1)A + (y-2)B + (z+1)C = 0$
$(x-0)A + (y-1)B + (z+2) = 0$

$\cancel{A I J T B}$

eq^n of plane $\begin{vmatrix} x-1 & y-2 & z+1 \\ x & y-1 & z+2 \\ 1 & -2 & 2 \end{vmatrix} = 0$

Que: the feet of Normal from origin to the Plane is (1, 2, 3). find eqn of plane

(0,0,0) (1,2,3)

$$(x-1)A + (y-2)B + (z-3)C$$

$$(x-1)1 + (y-2)2 + (z-3)3 = 0$$

(1,2,3)

(0,0,0)

$$\frac{1}{\sqrt{14}}i + \frac{2}{\sqrt{14}}j + \frac{3}{\sqrt{14}}k$$

$= 0$ (14)

$$x + 2y + 3z = 14.$$

Que:

$$P_1 = x + 2y - 3z = 0$$
$$P_2 = 2x + y + z + 3 = 0$$

$$x + 2y - 3z = 0$$
$$2x + y + z + 3 = 0$$
$$\frac{1}{2} = \frac{2}{1}$$

(i) find angle b/w planes

Formula Based

$$\cos\theta = \frac{1.2 + 2.1 + -3\times1}{\sqrt{1+4+9}\cdot\sqrt{4+1+1}}$$

(ii) find dirn Ratio of line of intersection of Both plane

$\boxed{1, 2, -3 \quad 1+2-3 \quad \frac{1}{\sqrt{14}}i + \frac{2}{\sqrt{14}}j - \frac{3}{\sqrt{14}}k \quad \frac{2}{\sqrt{6}}i + \frac{1}{\sqrt{6}}j + \frac{1}{\sqrt{6}}k}$

- dc of line of intersection of plane =

$$\begin{vmatrix} i & j & k \\ 1 & 2 & -3 \\ 2 & 1 & 1 \end{vmatrix}$$

(iii) Find eq^n of plane passes throw line of intersection of plane P_1 and P^2 and also passes throw (1, 2, 1)

$\begin{vmatrix} i & j & k \\ 1 & 2 & -3 \\ 2 & 1 & 1 \end{vmatrix}$ ~~5i + 6j + 3k~~

$\begin{vmatrix} i & j & k \\ 5 & 6 & 3 \\ 1 & 2 & 1 \end{vmatrix}$

= 1 2 1

~~(x-1) i + (y-2)~~

$$P_1 + \lambda P_2 = 0$$

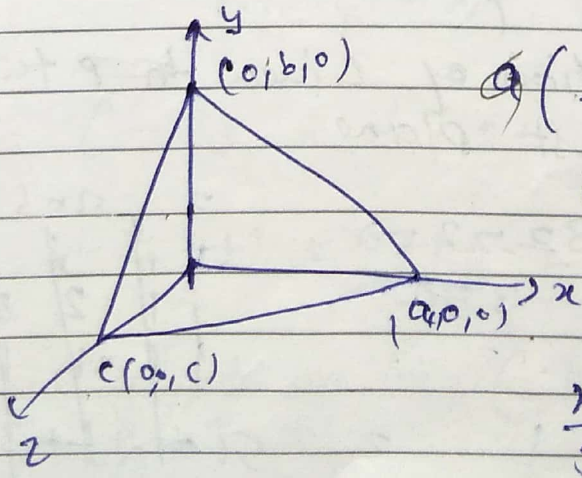
(1, 2, 1) $\quad x + 2y - 3z) + \lambda(2x + y + z + 3) = 0$

$$\lambda = -$$

$$P_1 + \lambda P_2 = 0$$

Que: A plane meet Quadrant axis A, B, C, such that Centroid of triangle A, B, C is Point (1, 2, 3) find eq^n of plane.

Ans : $(x-1)A + (y-2)B + (z-3)C = 0$



$$G\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$$

$\;\;\;\;\uparrow \;\; \uparrow \;\; \uparrow$
$\;\;\;\;1 \;\;\; 2 \;\;\; 3$

$\frac{x_1 + x_2 + x_3}{3} = 1$

$\frac{y_1 + y_2 + y_3}{3} = 2$

$\frac{z_1 + z_2 + z_3}{3} = 3$

$a = 3, \; b = 6, \; c = 9$

$$\frac{x}{3} + \frac{y}{6} + \frac{z}{9} = 1$$

Q. Find eq^n of plane passes through line of intersection of plane

$$x + 2y + 3z = 2$$
$$x - y + z = 3$$

at at a dist. $\frac{2}{\sqrt{3}}$ from $(3, 1, -1)$

Ans

$$x + 2y + 3z = 2$$
$$x - y + z = 3$$

$\frac{2}{\sqrt{3}} - \lambda$

$\sqrt{9+1+1}$

$= P_1 + \lambda P_2 = 0$

$$\left|\begin{array}{ccc} i & j & k \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{array}\right| = 0$$

$$(x + 2y + 3z - 2) + \lambda(x - y + z - 3) = 0$$

$$x(1+\lambda) + y(2-\lambda) + z(3+\lambda) - (2+3\lambda) = 0$$

$$\left|\frac{3(1+\lambda) + 1(2-\lambda) - 1(3+\lambda) - (2+3\lambda)}{\sqrt{(1-\lambda)^2 + (2-\lambda)^2 + (3+\lambda)^2}}\right| = \frac{2}{\sqrt{3}}$$

Ques $P_1 = x + 2y + 3z = 2$
$P_2 = x - y + z = 3$

Write eq^n of plane which passes through line of intersection of given ~~to P~~ two planes and $\perp$ to 1st plane

$$x + 2y + 3z - 2 = 0$$
$$x - y + z - 3 = 0$$

$= a \times b$

$$\left|\begin{array}{ccc} i & j & k \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{array}\right| = 0$$

$= 5i + 2j + 3k = 0$

3 given sum = dr

$(1+\lambda)\cdot 1 + (2-\lambda)\cdot 2 + (3+\lambda)\, 3 = 0$

$\lambda =$

① symmetrical form:
② un symmetrical form.

* Eq^n of line:

(1) Symmetrical form:

General pt. (x,y,z) on the line $\rightarrow \vec{r} = \vec{p} + \lambda \vec{q}$

passes through Pt $A(\vec{p})$ or $A(x_1, y_1, z_1)$

and dr/dc of line is $\vec{q}$

i.e $(a\hat{i} + b\hat{j} + c\hat{k})$

and λ is Parameter Here. for diff. pts on the line we have diff. λ

Vectorical form $x\hat{i} + y\hat{j} + z\hat{k} = (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$

$x = x_1 + a\lambda$ or

cartisian form $\boxed{\dfrac{x-x_1}{a} = \dfrac{y-y_1}{b} = \dfrac{z-z_1}{c} = \lambda}$

x, y, z, Coff. is 1 in mid - sign must.

Any pt. on the above line is

$P(x_1 + a\lambda,\ y_1 + b\lambda,\ z_1 + c\lambda)$

$\vec{r} = \hat{i} - 3\hat{j} + \lambda(2\hat{i} - \hat{j} + 3\hat{k})$ ~~the~~ p. $(1,-3,0)$

$\dfrac{(x-1)}{2} = \dfrac{y+3}{-1} = \dfrac{z-0}{3} = \lambda$

Line: any pt on it $(1+2\lambda,\ -3-\lambda,\ 3\lambda)$.

angle b/w x axis are dc
y axis
z axis

$\frac{x-1}{3} = \frac{y+4}{-1} = \frac{z-5}{0}$ ← Points ← Direction

$\bar{r} = (i-4j+5k) + \lambda(3i-j)$

* $A(x_1, y_1, z_1)$
& $B(x_2, y_2, z_2)$
line passes Through two Points

= eq^n: ~~$(x_2-x_1)i + (y_2-y_1)j + (z_2-z_1)k$~~

$$\boxed{\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} = \lambda}$$

$(0, \frac{7}{2}, \frac{11}{2})$

* eq^n of X-axis:

x axis
dc of x axis is (1, 0, 0)
dc " y " " (0, 1, 0)
" " z " " (0, 0, 1)

* eq^n of X-axis:

$$\frac{x-\alpha}{1} = \frac{y-0}{0} = \frac{z-0}{0}$$

eq^n of y axis: $\frac{x-0}{0} = \frac{y-\alpha}{1} = \frac{z-0}{0}$

eqⁿ of z-axis $\frac{x-0}{0} = \frac{y-0}{0} = \frac{z-\alpha}{1}$

~~z-~~ जिसके निचे zero है वह उस axis ~~point~~ को ⊥cular होगी

* $\frac{x-2}{3} = \frac{y+1}{2}$ & $z = 2$

Que represent in vectorical form

i.e $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-2}{0} = \lambda$ ⊥cular to z-axis

$\bar{r} = 2i \cancel{+} j + 2k + \lambda(3i + 2j)$

this line ⊥ to z-axis.

Q. Represent line in vectorical form and write any point on the line

$\frac{x-1}{-1} = \frac{1-x}{1} = \frac{2y+3}{4} = \frac{z-3}{-1} = \frac{x-1}{-1} = \frac{y+\frac{3}{2}}{2} = \frac{z-3}{-1} = \lambda$

$\bar{r} = i - \frac{3}{2}j + 3k + \lambda(-i + 2j - k)$.

$(1, -\frac{3}{2}, 3)$ line passes

dr of line $(-1, 2, -1)$

any pt on the line $(1-\lambda, -3/2 + 2\lambda, 3-\lambda)$

P + L

3D ⇒ Q-1 2, 3, 4, 5, 6, 7, 8, 9, 11, 12

Q. $\frac{3x+1}{1} = \frac{6y-2}{1} = \frac{3-z}{1}$

$\frac{x+\frac{1}{3}}{1/3} = \frac{y-\frac{2}{6}}{1/6} = \frac{z-3}{-1}$

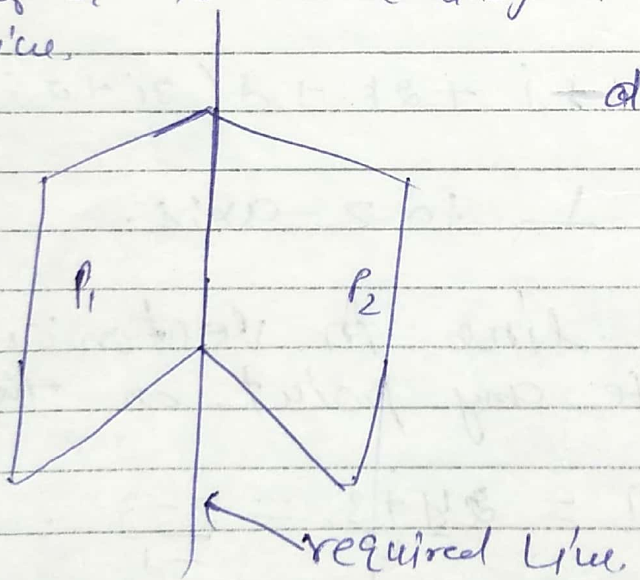
$-\frac{1}{3}i + \frac{2}{6}j + 3k + \lambda\left(\frac{1}{3}i + \frac{1}{6}j - k\right)$

$\left(-\frac{1}{3}, \frac{2}{6}, 3\right)$ Passes $dr = \left(\frac{1}{3}, \frac{1}{6}, -1\right)$

Unsymmetrical form of eqn of line:

$\overbrace{a_1x + b_1y + c_1z + d}^{P_1} = 0 = \overbrace{a_2x + b_2y + c_2z + d_2}^{P_2}$

This form of eqn is called unsymmetrical form of eqn of line.



dr of this line is ⊥cular to the normal of both plane Hence DR of required line

$$dr = \begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$i(b_1c_2 - c_1b_2) - j(a_1c_2 - a_2c_1) + k(a_1b_2 - a_2b_1)$

* Convert unsymetrical form of Line into Symetrical !

$$\frac{x-}{b_1c_2-c_1b_2} = \frac{y-}{a_2c_1-a_1c_2} = \frac{z-}{a_1b_2-a_2b_1}$$

Put $z=0$ (we can also put $x=0$ & $y=0$. any No.

we will put x, any of one.

$a_1x + b_1y + d_1 = 0$

$a_2x + b_2y + d_2 = 0$

$x = \alpha$

$y = \beta$

Now we have dr of the required.

to

$$\frac{x-\alpha}{b_1c_2-c_1b_2} = \frac{y-\beta}{c_2c_1-a_1c_2} = \frac{z-0}{a_1b_2-a_2b_1}$$

Q. Convert unsymetrical form of line.

$x-y+2z-3=0 = 2x+y-2z+4$. in symetrical form

$$\frac{x-(-\frac{1}{3})}{0} = \frac{y-(-\frac{10}{3})}{+6} = \frac{z-0}{3}$$

$$\begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ 2 & 1 & -2 \end{vmatrix}$$

$i(0)+j(+6)+3k.$

$+6j+3k.$

$z=0$

$x-y-3=0$

$2x+y+4=0$

$3x+1=0$

$3x = -\frac{1}{3}$

$z=0$

$-y-3=\frac{1}{3}$

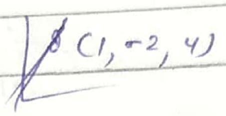
$-y = \frac{1}{3}+3$

$\cancel{\frac{1}{3}}+\frac{9}{3}$

$-y = \frac{10}{3}$ $y = -\frac{10}{3}$

Plane eq^n form left 3

Que) Find eq^n of line which passes throw (1, −2, 4) and equally inclined from +ve dir^n of all 3-axis.



$$\frac{(x-1)}{1} = \frac{(y+2)}{1} = \frac{(z-4)}{1} = \lambda$$

$$\cancel{\frac{x}{4-4y} = \frac{y+2}{2-4x} = \frac{z}{4+2x}}$$

$$\begin{vmatrix} i & j & k \\ 1 & 2 & 4 \\ x & y & z \end{vmatrix}$$

$(4-4y)i - (2-4x)j + (y+2x)k$

Que) Find dist. b/w lines

$$\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-3}{0}$$

$$\& \quad \frac{x+3}{4} = \frac{y-1}{-5} = \frac{z+2}{0}$$

$= 5$ **Ans**

$\hat{i} - \hat{j} + 3k + \lambda(2\hat{i} + \hat{j})$

$-3\hat{i} + \hat{j} - 2k + \lambda(4\hat{i} - 5\hat{j})$

$$\boxed{\begin{array}{l} \hat{i} - \hat{j} + 3k + \lambda(2\hat{i} + \hat{j}) \\ -3\hat{i} + \hat{j} - 2k + \lambda(4\hat{i} - 5\hat{j}) \\ D = \dfrac{-3-1-6}{\sqrt{1+1+9}\ \sqrt{9+1+4}} \quad \times \end{array}}$$

Q. Find point where line meet the plane

$$\frac{x+3}{2} = \frac{y-1}{-2} = \frac{z-2}{1} = \lambda$$

$x + 2y - z = 4.$ $\qquad x + 2y - z - 4 = 0$

$\cancel{(1, 4)}$

$-3\hat{i} + \hat{j} + 2k + \lambda(2\hat{i} - 2\hat{j} + k)$ ✗

$(-3 + 2\lambda, 1 - 2\lambda, 2 + \lambda)$ Any point on the line.

$$-3+2\lambda+2(1-2\lambda)-(-2+\lambda)=4$$
$$-3+2\lambda+2-4\lambda+2-\lambda=4$$
$$-3\lambda=3 \qquad (-5,3,-3)$$
$$\lambda=-1$$

* Line and Plane :

$$\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}=r$$

& $ax+by+cz+d=0$

Case -1 : Line is ⊥ to the Plane

In this Case Normal to plane and given line will be Parallel to each other

$$\frac{a}{l}=\frac{b}{m}=\frac{c}{n}$$

Case : 2

Line Parallel to the Plane

in this situation Normal to the plane is ⊥ cular to the line

$$a\alpha+b\beta+c\gamma+d\neq 0$$

$$al+bm+cn=0$$

(3) Line lie on the plane

$$al+bm+cn=0$$

Line passes through (α,β,γ) will satisfy
$$a\alpha+b\beta+c\gamma+d=0$$

Q. find angle b/w lines

$L_1 = 3x+2y+z-5=0=x+y-2z-3$

$L_2 : 2x-y-z=0=7x+10y-8z$

$\cancel{6+2+}$

$$\cancel{\frac{6-2-1}{\sqrt{1+1+4}\ \sqrt{49+100+64}}}$$

$$\cancel{\frac{90+63+17}{\sqrt{25+49+1}\ \sqrt{(18)^2+(9)^2+(17)^2}}}$$

$\bar{n}_1 = L_1 = \begin{vmatrix} i & j & k \\ 3 & 2 & 1 \\ 1 & 1 & -2 \end{vmatrix}$

$L_1 \quad -5i+7j+k$

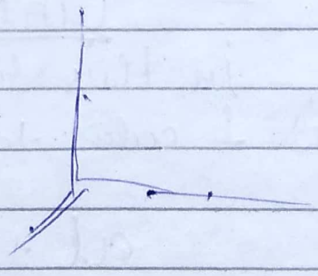
$\bar{n}_2 = L_2 = \begin{vmatrix} i & j & k \\ 2 & -1 & -1 \\ 7 & 10 & -8 \end{vmatrix}$

$= 18i+9j+27k$

$\cancel{\begin{vmatrix} i & j & k \\ -5 & 7 & 1 \\ 18 & 9 & 17 \end{vmatrix}} = 0$

$$\cos\theta = \frac{n_1 \cdot n_2}{|n_1|\,|n_2|} = 0 \qquad \theta = 90^\circ$$

Q. find points in which the line $x = 1+2t,\ y = -1-t$ & $z = 3t$ meet the coordinate plane i.e xy, yz, zx plane.



on xy -	$-1-1$,	$-t-2t$
yz	1,	$3t+t$
zx		X

Line cut xy-plane at $(z=0$ i.e $t=0) \Rightarrow (1,-1,0)$

xz " $(y=0$ i.e $t \Rightarrow (-1,0,-3)$

yz " $($

Q. Find eq^n of ~~plane~~ line through ~~poi~~ $(1, 4, -2)$ and parallel to planes

$$6x+2y+2z+3=0 \quad \& \quad \cancel{6+8}$$

$$x+2y-6z+4=0$$

$\cancel{6+4-12}$

~~$(1+4)x + (2+2)y + (6-12)z + 1 = 0$~~

dr of line $= \begin{vmatrix} i & j & k \\ 6 & 2 & 2 \\ 1 & 2 & -6 \end{vmatrix}$

$= -16i + 38j + 10k.$

$$\frac{x-1}{-16} = \frac{y-4}{38} = \frac{z+2}{10}$$

$(1, 0, -3)$

Q. Find dist. of point A from plane measured parallel to the line L:

$P: x - y - z = 9 \qquad A(1, 0, -3)$

$$L: \frac{x-2}{2} = \frac{y+2}{3} = \frac{z-6}{-6}$$

$2i - 2j + 6k + \lambda(2i + 3j - 6k).$

$= \cancel{-3i + 4j + k}$

$\begin{vmatrix} i & j & k \\ 1 & -1 & -1 \\ 1 & 0 & 3 \end{vmatrix}$

$\cancel{-3i - 4j + k}.$

$$\frac{x-1}{2} = \frac{y-0}{3} = \frac{z+3}{-6}$$

Any point $(1+2\lambda,\ 3\lambda,\ -3-6\lambda)$

$(1+2\lambda) - 3\lambda + 3 + 6\lambda = 9$

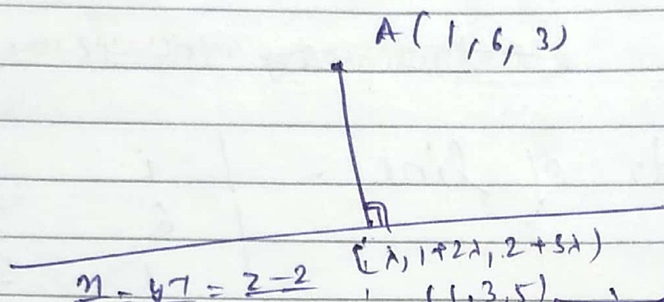
$\lambda = 1$

$= (3, 3, -9)$

Que: Find ⊥ dist of point $A(1,6,3)$ from the line

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

Also find image of A w.r.t the given line.



$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

$\hat{j} + 2\hat{k} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

$\lambda + \quad 2\lambda+2 \quad 3\lambda+2$

$$\frac{x-1}{1} = \frac{y-6}{2} = \frac{z-3}{3} = \lambda$$

$$\frac{1}{1} = \frac{6-1}{2} = \frac{3-2}{3} = \lambda$$

Image = $I(\alpha, \beta, r)$

$\frac{\alpha+1}{2} = 1, \frac{\beta+6}{2} = 3,$

$\frac{r+3}{2} = 5$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 6 & 3 \\ 0 & 1 & 2 \end{vmatrix} \quad 9\hat{i} - 2\hat{j} + 1\hat{k}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 6 & 3 \\ 1 & 2 & 3 \end{vmatrix} = 12\hat{i} + 4\hat{k}$$

dr of AM = $\langle \lambda-1, 2\lambda-5, 3\lambda-1 \rangle$

dr of line $\langle 1, 2, 3 \rangle$

AM ⊥ line

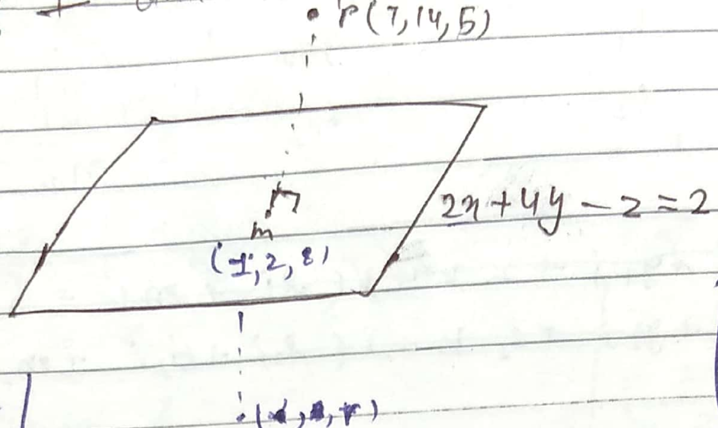
$$(\lambda - 1) + (2\lambda - 5)\cdot 2 + (3\lambda - 1)\cdot 3 = 0$$

$$14\lambda = 14$$

$$\lambda = 1$$

$$AM = \sqrt{(1-1)^2 + (3-6)^2 + (5-3)^2}$$

Que:- find foot of ⊥ drawn from Point $P(7,14,5)$ to the Plane



$$\begin{vmatrix} i & j & k \\ 2 & 4 & -1 \\ 7 & 14 & 5 \end{vmatrix} = 34i - 17j.$$

$$\begin{vmatrix} i & j & k \\ 7 & 14 & 5 \\ 2 & 4 & -1 \end{vmatrix} = -34i + 17j$$

$$\frac{x-7}{2} = \frac{y-14}{4} = \frac{z-5}{-1} = \lambda$$

$$(7+2\lambda,\ 14+4\lambda,\ 5-\lambda)$$

$$2(7+2\lambda)+4(14+4\lambda)-(5-\lambda)=2$$

$$\lambda = -3$$

Q in above Que. find image of Point P w.r.t to given Pla

$$\frac{\alpha+7}{2}=1 \qquad \frac{14+\beta}{2}=2$$

$$\frac{5+\gamma}{2}=8.$$

Que: if lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m} = \frac{z-z_1}{n_1}$ &

$$\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_1}$$

two lines are coplanar / Intersect the P.t

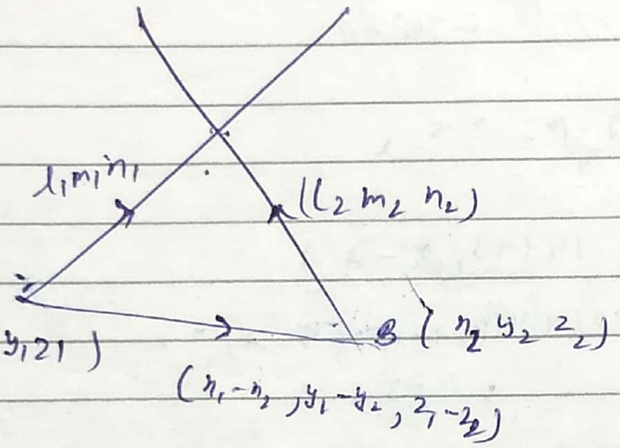
~~$x-x_2$~~

$$\boxed{\begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0}$$

~~$L_1 = x_1 i + y_1 j + z_1 k + \lambda(l_1 i + m_1 j + n_1 k)$~~

~~$L_2 = x_2 i + y_2 j + z_2 k + \lambda(l_2 i + m_2 j + n_2 k)$~~

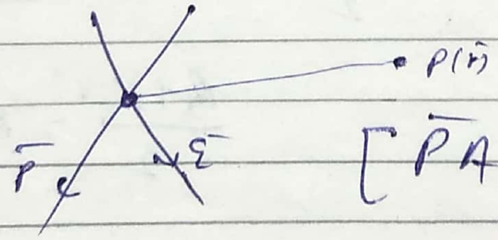
$= \cancel{\quad}$



Two intersecting lines determine unique plane

$\bar r = \bar a + \lambda \bar p$

$\bar r = \bar a + \lambda \bar q$



$[\bar{PA}\ \ \bar p\ \ \bar q] = 0$

M-1

$[\bar r - \bar a \quad \bar p \quad \bar q]$

or: $\bar{PA} = \lambda \bar p + \mu \bar q$

$$\boxed{\bar r = \bar a + \lambda \bar p + \mu \bar q}$$

Q. $\bar{r} = i + j + \lambda(2j - k) + \mu(i + 2j + k)$

Express plane in Cartesian form.

$= \quad i - 2j -$

$=$

$\bar{a}i$

$$= \begin{vmatrix} i & j & k \\ 0 & 2 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$4i + j - 2k$.

Plane passes through A

vector line on the plane

$$\boxed{\bar{r} = \bar{a} + \lambda p + \mu \bar{q}}$$

$(x-1)4 + (y-1)(-1) + (z-0)(-2) = 0$

Que: If lines are coplanar $\frac{x}{a_1} = \frac{y}{b_1} = \frac{z}{c_1}$

$\frac{x}{a_2} = \frac{y}{b_2} = \frac{z}{c_2}$, $\frac{x}{a_3} = \frac{y}{b_3} = \frac{z}{c_3}$

then find condition

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

~~$a_1 i + b_1 j + c_1 k$~~

$a_2 i + b_2 j + c_2 k$.

$a_3 i + b_3 j + c_3 k$.

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

Naturally

Q.2 $x = 5$, $\frac{y}{3\alpha} = \frac{z}{-2}$ & $x = \alpha$, $\frac{y}{-1} = \frac{z}{2-\alpha}$

if these two lines are coplanar then find α

$$= \begin{vmatrix} 5 & 3\alpha & -2 \\ & & \end{vmatrix} =$$

~~$5j$~~ 3α, $5i + 3\alpha j + -2k$.

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$$\frac{x-5}{0} = \frac{y}{3-\alpha} = \frac{z}{-2} \quad \& \quad \frac{x-\alpha}{0} = \frac{y}{-1} = \frac{z}{2-\alpha}$$

$A(5,0,0)$ dr of line AB $\langle 5-\alpha, 0, 0\rangle$

$B(\alpha,0,0)$ dr of Ist line $\langle 0, 3-\alpha, -2\rangle$

II nd $\langle 0, -1, 2-\alpha\rangle$

$$\begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$$

Que: $\frac{x+1}{3} = \frac{y-2}{m} = \frac{z+3}{-2}$

$x - 3y + 6z + 7 = 0$

find m for which line is ||el to the Plane

$-i + 2j - 3k + \lambda(3i + mj - 2k) = 0$

$x - 3y + 6z = -7$

$$\begin{vmatrix} i & j & k \\ 3 & m & -2 \\ 1 & -3 & 6 \end{vmatrix}$$

$= (6m - 6)i +$

$$\begin{vmatrix} i & j & k \\ -1 & 2 & -3 \\ 1 & -3 & 6 \end{vmatrix}$$

$= 3i - 9j + k$

Normal to the plane is ⊥r to given line

$1 \cdot 3 + m(-3) + (-2) \cdot 6 = 0$

$m = -3.$

Q. find a, b for which the line

$$\frac{x-2}{a} = \frac{y+3}{4} = \frac{z-6}{-2}$$

$\frac{3}{a} = \frac{-2}{4} = \frac{6}{-2}$

$2a = 3b$

is $\perp$ to the plane

$$3x - 2y + bz + 10 = 0$$

~~$3x - 2y + bz = -10$~~

~~$2i - 3j + 6k + \lambda(ai + bj + 2k)$~~

In this case
Line will be $\parallel$ to the
normal to the plane

$$\begin{vmatrix} i & j & k \\ a & b & -2 \\ 3 & -2 & b \end{vmatrix} = 0$$

$= (b^2 - 4)i + ab -$

$$\frac{a}{3} = \frac{4}{-2} = \frac{-2}{b}$$

$$a = 6 \quad b = 1$$

Ques: find eq of Straight Line (A (1, 2, 3))
(i) $\parallel$ to z-axis.

A (1, 2, 3)

$$\frac{x-1}{0} = \frac{y-2}{0} = \frac{z-3}{1}$$

(ii) $\perp$ Cul to z - axis

$$\frac{x-1}{l} = \frac{y-2}{m} = \frac{z-3}{0}$$

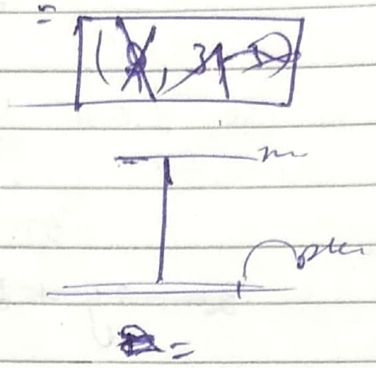
H.W J.H.

Line In the Plane

Que! find eq^n of plane through the line

$$\frac{x-2}{2} = \frac{y-3}{3} = \frac{z-4}{5}$$

and Parallel to x - axis



$= (x-2)A + (y-3)B + (z-4)C$.

$\cancel{\frac{x-2}{2} = \frac{y-3}{0} = \frac{z-4}{0}}$

$\cancel{(x-2)2 + (y-3)3 + (z-4)5}$

Normal to the plane is $\langle A, B, C \rangle \perp$

it is $\perp^r$ x - axis $(1, 0, 0)$

$A \cdot 1 + B \cdot 0 + C \cdot 0 = 0$

$(y-3)B + (z-4)C = 0$

$0 \cdot 2 + B\,3 + C - 5 = 0$

$B = \frac{5}{3}C$

$(y-3)B + (z-4)C = 0$

$-5(y-3) + 3(z-4) = 0$

M-2 $\quad (x-2)A + (y-3)B + (z-4)C = 0$

$A \cdot 1 + 3 \cdot 0 + C \cdot 0 = 0$

$2A + 3B + 5C = 0$

$$\begin{vmatrix} x-2 & y-3 & z-4 \\ 1 & 0 & 0 \\ 2 & 3 & 5 \end{vmatrix} = 0$$

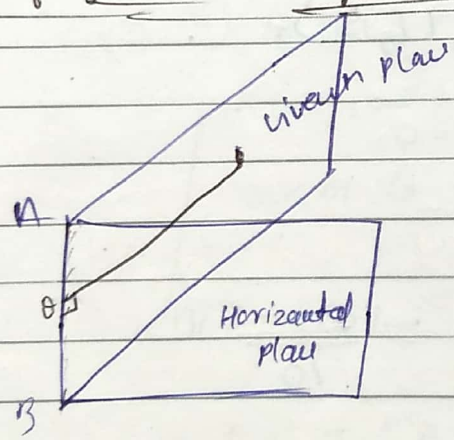
* angle b/w line $\bar{r} = \bar{p} + \lambda \bar{q}$ & plane $\bar{r}\cdot\bar{n} = a$. $= \theta$

$\therefore$ angle b/w normal to the Plane & line $= 90-\theta$

$\therefore \bar{n}\cdot\bar{q} = |\bar{n}||\bar{q}| \cos(90-\theta)$

$$\sin\theta = \frac{\bar{n}\cdot\bar{q}}{|\bar{n}||\bar{q}|}$$

* Line of greatest slope in plane :-



It is a Line in given plane and ⊥ to the line of intersection of given plane with Horizontal Plane Hence PQ is the line of greatest slope. Its d/c can be determine that fact that

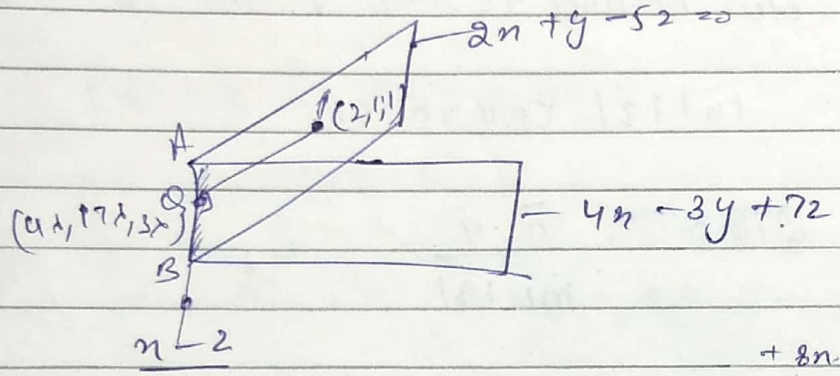
i) it lies in the g plane

ii) it is ⊥ to the AB i.e intersection of g and H Planes.

* DR of line AB is cross product of Normal to Horizontal and given Plane

Que:- if H plane → $4x - 3y + 7z = 0$ then find eqn of line of Greatest slope throw the point $P(2,1,1)$ in given plane

$2x + y - 5z = 0$



$$\begin{vmatrix} i & j & k \\ 2 & 1 & -5 \\ +4 & -3 & 7 \end{vmatrix}$$

$= +8i + 34j + 10k$

$\langle 8i + 34j + 10k \rangle$

$+8x + 36j + 10k$

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dr of line AB $\langle 4, 17, 5 \rangle$.

$4x - 3y + 7z = 0$
$2x + y - 5z = 0$

$$\frac{x-0}{4} = \frac{y-0}{17} = \frac{z-0}{10}$$

$PQ \perp AB$

dr of PQ $\langle 4\lambda - 2, 17\lambda - 1, 5\lambda - 1 \rangle$

$(4\lambda - 2)4 + (17\lambda - 1) \cdot 17 + (5\lambda - 1) \cdot 5 = 0$

$$\lambda = \frac{1}{11}$$

Q. Find eqn of plane which contains two parallel lines

$$\frac{x-4}{1} = \frac{y-3}{-4} = \frac{z-2}{5}$$

&

$$\frac{x-3}{1} = \frac{y+2}{-4} = \frac{z}{5}$$

Ans
$$\begin{vmatrix} x-4 & y-3 & z-2 \\ x-3 & y+2 & z \\ 1 & -4 & 5 \end{vmatrix} = 0$$

~~$x-4(5(y+2)+4z) - y-3(5(x-3)-z) + z-2(-4(x-3) - y+2))$~~

$A(4,3,2)$; $B(3,-2,0)$

$(x-4)A + (y-3)B + (z-2)C = 0$

$A(4,3,2)$

dr of line AB $\langle 1,5,2\rangle$ is $\perp^r$ to Normal to plane

$B(3,-2,0)$

$1A + 5B + 2C = 0$

$A \cdot 1 + B(-4) + C5 = 0$

$$\begin{vmatrix} x-4 & y-3 & z-2 \\ 1 & 5 & 2 \\ 1 & -4 & 5 \end{vmatrix} = 0$$

Revia circle

Que: P.T Three Planes

$$2x + y - 4z - 17 = 0$$
$$3x + 2y - 2z - 25 = 0$$
&
$$2x - 4y + 3z + 25 = 0$$

intersect at a point. find the co-ordinates of the point.

$$\begin{vmatrix} i & j & k \\ 2 & 1 & -4 \\ 3 & 2 & -2 \end{vmatrix} = 4i - 2j + j$$

$2i + j - 4k = 17$

$$\begin{vmatrix} 2 & 1 & -4 \\ 3 & 2 & -2 \\ 2 & -4 & 3 \end{vmatrix} \neq$$

$\cancel{7 = 47}$

$2x + y - 4z = 17$

$$\begin{vmatrix} 2 & 1 & -4 \\ 3 & 2 & -2 \\ 2 & -4 & 3 \end{vmatrix} = 47 \neq 0$$

$-4 - 13 + 64$

$$2(6-8) - 1(9+4) + 4(-12-4)$$
$$2(-2) - (13) + 4(-16)$$
$$-4 - 13 + 64$$
$$-17 + 64 = 47 \neq 0$$

SBG STUDY